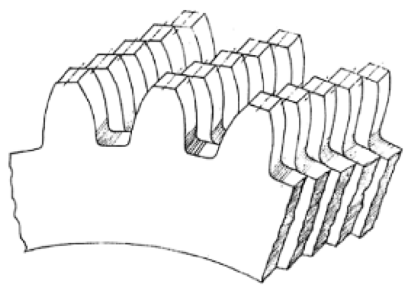
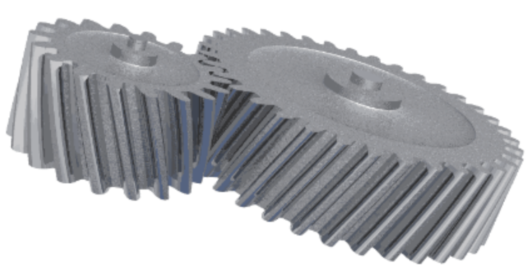
Helical gears are a refined form of spur gears, where the teeth are cut at an angle (called the helix angle) to the gear axis, resulting in a gradual engagement of teeth. This angular orientation causes the gear teeth to appear as if they are part of a helix or spiral around the gear face. In effect, a helical gear resembles a stack of thin spur gears, each slightly rotated to produce the desired helix angle. The teeth contact progressively along the face, leading to smoother and quieter operation compared to spur gears.



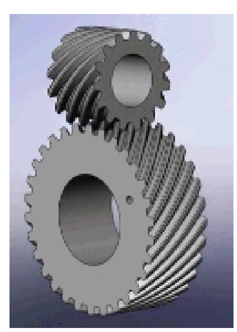
Structurally, helical gears are constructed by approximating them as a composite of stepped spur gears, laminated with infinitesimal thickness and a slight angular phase difference. This configuration results in a continuous and overlapping tooth engagement, which minimizes vibration and allows higher load transmission with reduced noise levels. This makes them highly suitable for high-speed and heavy-duty applications.

Unlike spur gears, where the teeth engage all at once, helical gear teeth engage gradually due to their angled orientation. This results in smoother operation, quieter running, and higher load-carrying capacity. Because the engagement is progressive, the load is distributed more evenly along the tooth surface, reducing the bending stress on each tooth. There are two forms of helical gears: The parallel helical gears and the crossed helical gears.



Parallel Helical Gears

Helical gears used to transmit motion or power between parallel shafts.



Crossed Helical Gears

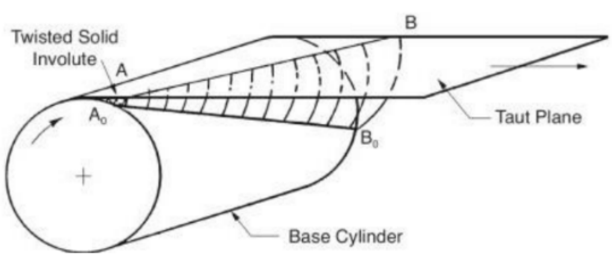
Helical gears used to transmit motion or power between nonparallel shafts.

In helical gears, the line of contact is diagonal, not straight across as in spur gears. As a result, the contact starts at one end of the tooth and progresses along its length, allowing for greater strength and smoother motion. This also allows for higher operating speeds and lower noise levels, which is why helical and herringbone gears are commonly used in heavy-duty gearboxes and high-speed machinery.

The helix angle, typically between 15° and 25°, plays a significant role in gear performance. A larger helix angle increases the tooth overlap and smoothness of operation but also introduces an axial thrust load—a side force that must be absorbed by bearings.

Tooth Profile and Geometry

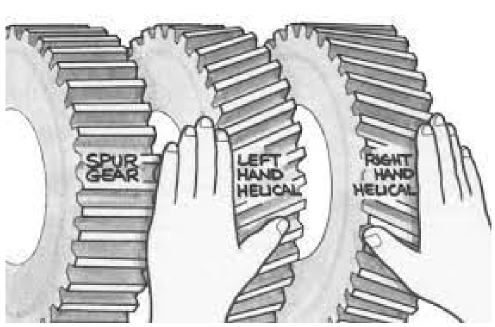
The **tooth profile** of a helical gear is typically an involute in the plane of rotation, just like in spur gears. However, the helical tooth extends diagonally across the gear face, resulting in an elongated contact area during operation.



The generation of the helical involute tooth can be visualized by wrapping a taut string around a cylinder and tracing a curve, similar to the spur gear, but in a twisted form along the cylinder's axis.

Hand of the gear

Helical gears are classified as right-hand or left-hand, depending on the direction of the helix when viewed from the gear face.

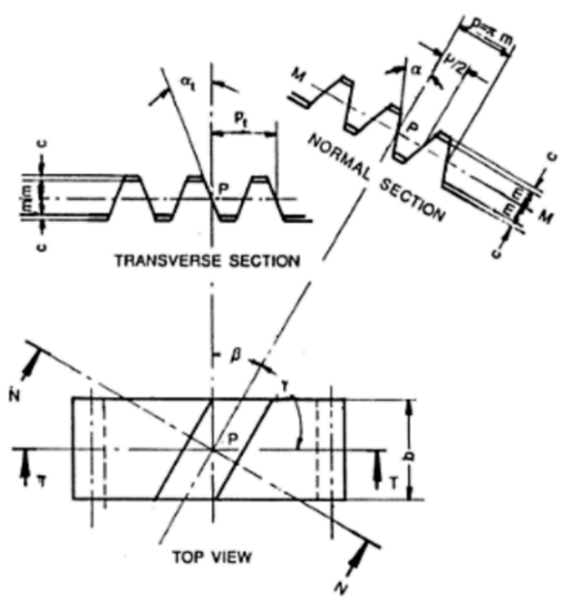


Because of their geometry, helical gears are not interchangeable unless their helix directions match appropriately—right-hand gears mesh with left-hand gears on parallel shafts. For optimal operation, the face width of a helical gear should be large enough to support the axial tooth contact, typically satisfying the relation $b \geq 2P_a$, where P_a is the axial pitch.

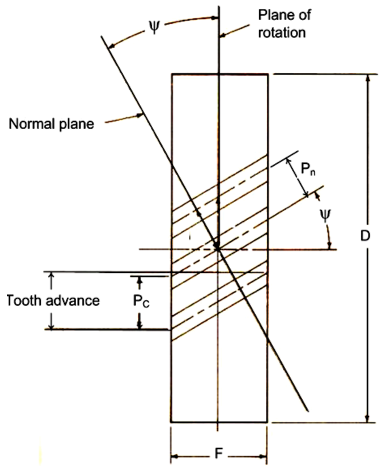
Fundamentals of Helical Teeth

Reference: Design of Machine Elements 4th Edition by Virgil Faires page 397-405

Helical gear teeth can be described using the same basic terminology as spur gears, but in both normal and transverse planes:



- Normal Plane: Plane perpendicular to the helix, which is used to define the normal pitch, normal module, and normal pressure angle.
- Transverse Plane: Plane of rotation where parameters like transverse pitch, transverse module, and base pitch are defined.



Definitions:

Face width, f

For average practice: $f \geq \frac{2\,P_C}{tan\,\psi}$

For significant advantage from overlapping of tooth engagement, (ψ=15° to 25°)

$$f \geq 2\,P_a$$

Normal circular pitch, P_{Cn}

The distance between the teeth measured on the pitch surface along a normal to the helix.

$$P_{Cn} = P_C\,cos\,\psi = \pi\,m_n$$

Where: ψ = helix angle
m_n = module in the normal plane
P_C = transverse circular pitch

Normal diametral pitch, P_{dn}

The number of teeth per inch of pitch diameter as measured in the normal plane, which is the plane perpendicular to the direction of the gear teeth

$$P_{dn} = \frac{P_d}{cos\,\psi}$$

Where: P_d = transverse diametral pitch
$P_{dn}P_{Cn} = \pi$

Normal pressure angle, ϕ_n

The angle between the tooth profile and a radial line to the pitch circle, measured in the normal plane—the plane oriented perpendicular to the direction of the helical gear teeth.

$$tan\,\phi_n = tan\,\alpha\,cos\,\psi$$

Where: ϕ = pressure angle

Axial pitch, P_a

The distance measured parallel to the gear axis between corresponding points on adjacent teeth along the pitch cylinder.

$$P_a = \frac{P_C}{\tan \psi}$$

Center distance, c

$$c = \frac{D_1 + D_2}{2}$$

Velocity ratio, VR

$$VR = \frac{N_1}{N_2} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

Axial load or thrust load, F_a

The force component in a helical gear system that acts parallel to the shaft axis, tending to push the gear along the shaft.

$$F_a = F_t \tan \psi$$

Where:

F_t = transmitted or tangential load. This represents the actual useful force delivered by the gear teeth to perform work. It is derived directly from the required horsepower and the pitch line velocity of the gears.

$$HP = \frac{F_t v_m}{33{,}000}$$

Dynamic load for helical gears, F_d

The dynamic load is the total effective force acting on the gear teeth during operation.

Buckingham's equation:

$$F_d = F_t + \frac{0.05 v_m \left(F_t + C \cdot f \cos^2\psi\right) \cos \psi}{0.05 v_m + \sqrt{\left(F_t + C \cdot f \cos^2\psi\right)}} \quad ; \quad \text{lbs}$$

C = Use Table AT 25, Values of C For e = 0.001 in, page 601: Design of Machine Elements 4th Edition by Virgil Faires.

The deformation factor C represents the stiffness and elastic properties of the materials used in the gear set. It accounts for how much the gear teeth will deflect under a given load and is dependent on the modulus of elasticity and the manufacturing error (e).

$v_m = \pi D_p N_p$ = Pitch line velocity is the linear speed of the gears at the pitch circle, typically measured in feet per minute (fpm). It is a critical variable in the Buckingham equation because higher velocities increase the impact forces between teeth.

f = Face width, in inches

F_t = transmitted or tangential load in lbs

Limiting wear load:

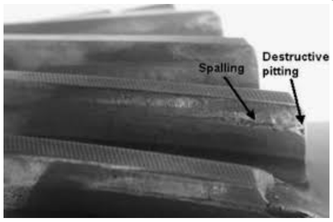
The limiting wear load defines the maximum capacity of the gear tooth surfaces to resist pitting and surface fatigue over millions of cycles. It is a function of the gear geometry, the curvature of the teeth, and the hardness of the materials.

$$F_W = \frac{f D_1 Q K_g}{\cos^2 \psi} \quad ; \quad \text{lbs}$$

$$Q = \frac{2T_g}{T_g + T_p} = \frac{2(VR)}{(VR) + 1}$$

K_g = Use Table AT 26, Values of Limiting Wear-Load Factor, page 603: Design of Machine Elements 4th Edition by Virgil Faires.
This is a material-dependent factor that reflects the surface fatigue strength of the gear-and-pinion combination. It is primarily determined by the Brinell Hardness Number (BHN) of the materials and the pressure angle.

Q is a geometric factor used in the wear equation to account for the relative curvature of the meshing teeth. It is determined by the number of teeth on the pinion and the gear (or the velocity ratio).

$F_W \geq F_d$	The gear set can operate continuously for its intended life without suffering from surface failure.
$F_W < F_d$	The gear set is in a state of Surface Durability Failure. This means the "demand" (the dynamic forces hitting the teeth) exceeds the "capacity" (the hardness and fatigue strength of the tooth surfaces). Because the contact stress between the teeth is higher than the material's surface endurance limit, microscopic cracks form just below the surface of the metal. As the gears continue to rotate, these cracks grow until small, pit-like chunks of metal break away from the tooth profile. 

Strength of Helical Teeth (Lewis Equation)

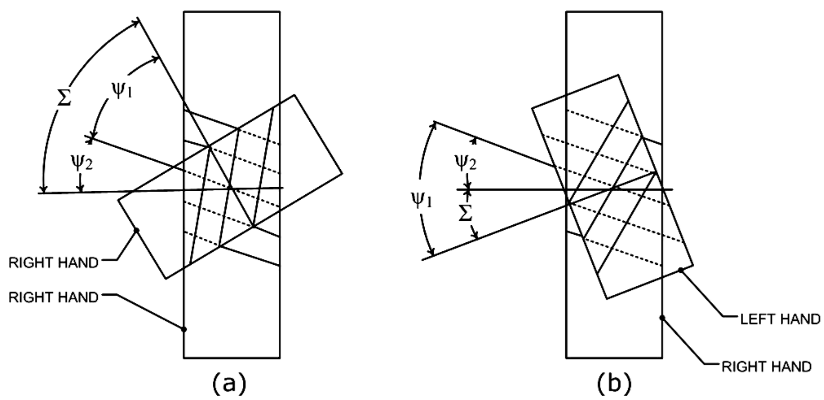
$$F_S = \frac{S_n f Y \cos\psi}{K_f P_d} = \frac{S_n f Y}{K_f P_{dn}} \quad ; \qquad \text{lbs}$$

F_s (Endurance Strength), often referred to as the static or structural strength of the gear, represents the maximum bending load that a gear tooth can withstand at its weakest point—the root fillet—without suffering from fatigue failure or permanent breakage.

Crossed Helical Gears

Crossed helical gears, also known as spiral gears, are a type of helical gear arrangement used when the shafts are non-parallel and non-intersecting, most commonly at a right angle. Unlike parallel-shaft helical gears that engage smoothly along a tooth line, crossed helical gears experience point contact between teeth. This results in a sliding motion rather than rolling, much like a worm gear rubbing across a mating surface.

Because of this sliding contact, rubbing and friction are greater, which limits the load-carrying capacity of crossed helical gears. These gears are thus better suited for light loads at moderate speeds. However, running-in under proper lubrication creates a polished line along the tooth contact, which can improve efficiency and reduce wear**.**



The angle between the shafts, denoted by Σ, is typically **90°**, and the gear pair must be carefully designed to maintain **equal normal pitch** and **pressure angle**.

When the gears are of the same hand

Angle between shafts, (figure a) $\Sigma = \psi_2 + \psi_1$

When the gears are of opposite hand

Angle between shafts, (figure b) $\Sigma = \psi_2 - \psi_1$

Center distance, C

$$C = \frac{D_2 + D_1}{2}$$

Velocity ratio, VR

$$VR = \frac{N_1}{N_2} = \frac{D_2 \cos \psi_2}{D_1 \cos \psi_1} = \frac{T_2}{T_1}$$

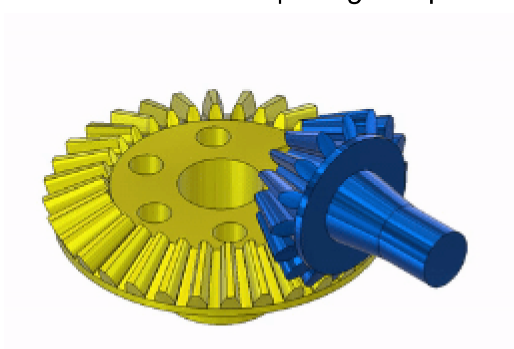
HERRINGBONE GEAR

Herringbone gears, also known as double-helical gears, are designed by combining a right-hand and a left-hand helix on the same gear. This configuration effectively cancels out axial thrust, which is a common issue in single helical gears. As a result, herringbone gears do not require thrust bearings to counteract side forces, making them ideal for heavy-duty applications.

The helix angles used in herringbone gears are typically large, ranging from **30° to 45°**, ensuring smooth, quiet, and efficient power transmission. To ensure equal load sharing between both helices, one of the shafts is allowed to "float" in the axial direction. The minimum **face width** of a herringbone gear is often governed by the relation $b \geq 4P_a$, where P_a is the axial pitch. Note that the center groove (necessary for tool clearance in some manufacturing methods) should not be counted in the active face width.

Herringbone gears can be made in one continuous piece or as two separate helices joined at the center. They are commonly used in **marine gearboxes**, **turbine drives**, and **steel mill equipment** due to their durability and high torque capacity. The gear teeth are usually cut after heat treatment, with hardness levels between **210 and 300 Brinell**, while pinions are made slightly harder for wear resistance—often 40 to 50 Brinell points higher.

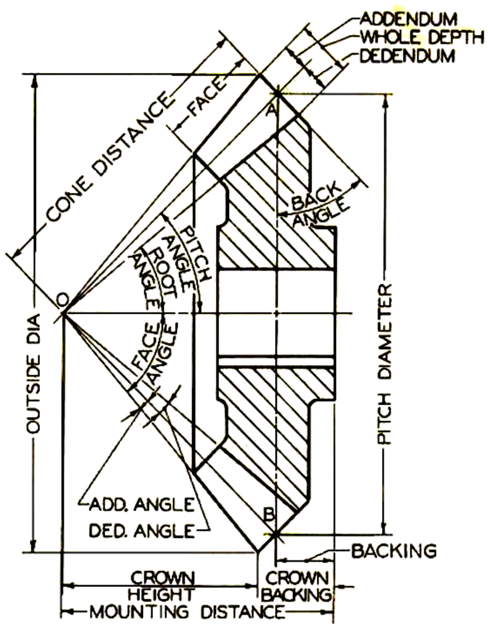
Bevel gears are designed to transmit motion between **intersecting shafts**, typically arranged at a 90° angle, though other angles are also possible. The teeth of bevel gears function similarly to those of spur and helical gears, enduring transmitted loads as well as dynamic forces caused by inaccuracies in tooth spacing and profile.



The key concern in bevel gear design is the **surface compressive stress**, which determines their wear resistance. Due to their geometry, bevel gears are not interchangeable like spur gears and are therefore always **designed and manufactured in matched pairs**.

Fundamentals of Bevel Gears

Reference: Design of Machine Elements 4th Edition by Virgil Faires page 407-425



The geometry of bevel gears is based on a **pitch cone** rather than a pitch cylinder, as is the case with spur gears. This cone tapers toward a point known as the **cone center**, where the pitch cones of mating gears ideally intersect.

Where: Triangle AOB is the pitch cone

The circular pitch and diametral pitch are defined in the same manner as for spur gears.

Pitch angle, γ

If the angle between shaft, $\Sigma = 90^o$:

$$\gamma_1 = tan^{-1}\left(\frac{N_2}{N_1}\right) = tan^{-1}\left(\frac{D_1}{D_2}\right) = tan^{-1}\left(\frac{T_1}{T_2}\right) \quad \text{(pinion)}$$

$$\gamma_2 = tan^{-1}\left(\frac{N_1}{N_2}\right) = tan^{-1}\left(\frac{D_2}{D_1}\right) = tan^{-1}\left(\frac{T_2}{T_1}\right) \quad \text{(gear)}$$

If the angle between shaft, $\Sigma < 90^o$:

$$\gamma_2 = tan^{-1}\left[\frac{sin\,\Sigma}{\frac{T_1}{T_2} + cos\,\Sigma}\right]$$

$$\gamma_1 = \Sigma - \gamma_2$$

Cone distance, L

The slant height of the pitch cone, hence the same for both gear and pinion.

$$L = \frac{D}{2 \sin \gamma}$$

Outside diameter, D_O

$$D_O = D + 2a \cos \gamma$$

Where: D = pitch diameter

Addendum angle, α

$$\alpha = tan^{-1} \frac{a}{L}$$

Where: a = addendum

Dedendum angle, δ

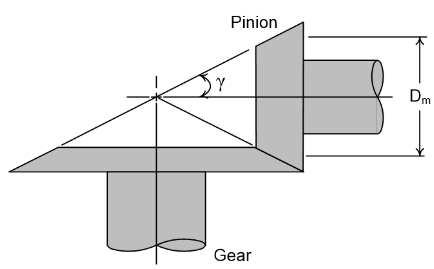
$$\alpha = tan^{-1} \frac{b}{L}$$

Where: b = dedendum

Velocity Ratio, VR

$$VR = \frac{N_1}{N_2} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

Mean diameter, D_m



$$D_m = D - f \sin \gamma$$

Where: f = face width

Proportions of long- and short-addendum bevel gears:

A long addendum pinion and a short addendum gear have more action in recess than approach, have strong pinion teeth and can have a lower pressure angle without undercut.

	Pinion (driver)	**Gear (driven)**
Addendum	$\frac{1.4}{P_d}$	$\frac{1.6}{P_d}$
Dedendum	$\frac{1.7571}{P_d}$	$\frac{1.5571}{P_d}$
Tooth thickness on the pitch line		
14.5°	$\frac{1.778}{P_d}$	$\frac{1.364}{P_d}$
20°	$\frac{1.862}{P_d}$	$\frac{1.280}{P_d}$

DESIGN OF BEVEL GEARS

Dynamic Load for Generated Bevel Gears (F_d)

In bevel gear design, the **Dynamic Load (F_d)** represents the total force acting on the teeth during operation, accounting for both the power transmitted and the additional forces caused by velocity, mounting conditions, and service environment.

The dynamic load is calculated using the AGMA (American Gear Manufacturers Association) approach as follows:

$$F_d = (VF)N_{sf}K_mF_t$$

Where: F_t = transmitted load, the tangential force derived from the power and speed

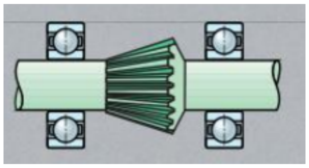
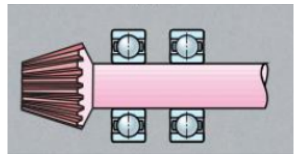
$$F_t = \frac{33{,}000\ (HP)}{v_m}$$

v_m = (Pitch-line Speed); Measured at the large end of the bevel gear.

$$v_m = \pi D_p N_p$$

K_m = Load Distribution Factor, Bevel gears are highly sensitive to misalignment and deflection because the teeth are tapered. The life of the gear increases with the rigidity of the mounting.

Gear Mounting

Straddle mounted means the gear is supported by bearings on both sides 	Overhung (cantilevered) means it is supported on only one side, which is less rigid.. 

Mounting Condition	**K_m Value**
Both gears straddle mounted	1.0 to 1.1
One gear straddle, one overhung	1.1 to 1.25
Both gears overhung	1.25 to 1.4

VF = Velocity Factor, the Velocity Factor accounts for the internal dynamic increment (impact forces) between teeth at high speeds.

For Commercial Straight Bevel Gears:

$$VF = \frac{50 + \sqrt{v_m}}{50}$$

For Commercial Spiral Bevel Gears:

$$VF = \left[\frac{78 + \sqrt{v_m}}{50}\right]^{1/2}$$

Speed Limits:

Straight and Zerol bevel gears are generally limited to pitch-line speeds below 1,000 fpm.

Spiral Bevel and Hypoid gears are recommended when speeds exceed 1,000 fpm or when the pinion rotates faster than 1,000 rpm.

N_{sf}= service factor, The service factor is a multiplier used to account for the type of power source (motor vs. engine) and the nature of the load (uniform vs. heavy shock). It ensures the gear can handle the specific environment of the application.

Rated Strength of Bevel Gears

This refers to the ability of the gear teeth to resist mechanical failure under load. Specifically, it focuses on the Bending Strength—the capacity of the tooth root to withstand the applied forces without snapping off like a cantilever beam.

$$F_S = \frac{S_d f J}{P_d} \cdot \frac{K_l}{K_S K_t K_r}$$

S_d = Design Flexural Stress, the allowable internal stress the material can handle before permanent deformation. Refer to page 414: Design of Machine Elements 4th Edition by Virgil Faires.
f = Face Width, the actual length of the gear tooth contact area along the axis, inches

J = Geometry Factor, a dimensionless number that accounts for the shape, pressure angle, and curvature of the bevel tooth. Figure 15.5 and 15.6 page 416. The J factor is dependent on the gear ratio, we must know the Number of teeth on the pinion, T_P and Number of teeth on the gear, T_G.

P_d = Diametral Pitch

K_l = Life Factor for strength, adjusts the strength based on how many cycles the gear will turn, refer to page 413.

K_S = Size Factor, a correction for larger teeth which have a higher statistical chance of material flaws, choose K_S, according to Pitch, P_d, refer to page 413.

K_t = Temperature Factor, adjusts for heat. It is usually 1.0 if operating below 160°F, refer to page 413.

K_r = Strength Reliability Factor, it determines the probability of the gear set surviving its intended life without a fatigue failure

- $K_r = 0.75$ (1 failure in 3): This is extremely low reliability (66% success rate). In this scenario, you are intentionally under-designing. This is almost never used in professional machinery unless the part is meant to be a "fuse" or is incredibly cheap and easy to replace.
- $K_r = 1.12$ (1 failure in 100): This represents 99% Reliability. This is the standard "commercial grade" baseline. It’s acceptable for general equipment where a failure once in a while is annoying but not life-threatening or financially catastrophic.

- $K_r = 1.50$ or more (High Reliability): This is the threshold for 99.99% Reliability (1 failure in 10,000). This is the standard for "Continuous Duty" industrial machines, speed reducers, and factory equipment. If a machine runs 24/7, you need this 1.5 buffer to account for metal fatigue over millions of cycles.
- $K_r = 3.0$ (Maximum Safety): This is used for critical systems where failure is not an option—such as elevators, aircraft flight controls, or medical equipment. Here, you are making the gear 3 times stronger than the average predicted load to ensure that even under the worst possible conditions, the gear will not break.

Rated Wear Load for Bevel Gears

Used to determine the maximum load a gear surface can withstand before it begins to fail due to **pitting** (surface fatigue). It is based on the Hertzian contact-stress theory, which calculates the pressure between two curved surfaces.

$$F_W = D_p f I \frac{S_{cd}^2}{C_e} \left(\frac{C_l}{K_t C_r} \right)^2$$

D_p = Pitch Diameter of Pinion, a larger diameter reduces the curvature of the teeth, allowing for more surface area and a higher load capacity.

f = Face Width, a wider gear spreads the force over a larger area, increasing the wear resistance linearly.

S_{cd} = Design Contact Stress, this is the "yield strength" of the surface. As shown in Table 15.3, page 415, it is highly dependent on the material's hardness. For Carburized Steel, this value is 225 ksi.

C_e = Elastic Coefficient, this factor accounts for the stiffness (Modulus of Elasticity) of the materials. Since steel is very stiff, the contact area is tiny, creating higher stress as shown in Table 15.4, page 415,. For steel-on-steel gears, this is typically 2800.

C_l = Life Factor for Wear, this adjusts the allowable load based on how many cycles the gear will turn, refer to page 415. For "indefinite life" (10^7 cycles or more), we use C_l=1.0.

K_t = Temperature Factor.

C_r = Wear Reliability Factor, it prevents the surface of the gear tooth from pitting or flaking.

- $C_r = 0.75$ (1 failure in 3): This represents Extremely Low Reliability (66% success rate). In this scenario, the gear surface is intentionally under-designed. This is almost never used in professional engineering because the gear surface would experience rapid pitting and wear-out. It is only considered if the gear is intended to be disposable or is very easy to replace.
- $C_r = 1.12$ (1 failure in 100): This represents 99% Reliability. This is the standard "Commercial Grade" baseline for wear. It is acceptable for general equipment where surface noise or minor pitting is annoying but does not pose a major safety risk or lead to catastrophic financial loss.
- $C_r = 1.25$ (1 failure in 1,000): This represents 99.9% Reliability (Highly Reliable). This is "Enhanced Industrial Grade." It is the practical limit for many high-quality industrial designs. It is significantly more durable than commercial grade, offering a 10x lower failure rate, but it stops just short of the extreme requirements for "High Reliability" (24/7 service).
- $C_r = 1.50$ or more (High Reliability): This is the threshold for 99.99% Reliability (1 failure in 10,000). This is the standard for "Continuous Duty" industrial machines and heavy-duty gearboxes. If a machine runs 24/7, this 1.5 buffer is required to ensure the tooth surface does not pit or flake over millions of cycles.
- $C_r = 3.0$ (Maximum Safety): This is used for Critical Systems where any surface damage is strictly prohibited—such as aerospace applications, nuclear drives, or medical life-support equipment. Here, you are ensuring that the surface hardness and finish are sufficient to remain smooth even under the most extreme pressure and operating conditions.

Precision Requirements:

If the pitch-line speed exceeds 8,000 fpm, the teeth must be ground after hardening to ensure accuracy and minimize noise.

Hypoid Advantages:

Hypoid pinions are typically larger than spiral-bevel pinions of the same capacity, allowing for the use of larger, stiffer shafts.

HYPOID GEARS

Hypoid gears are a special type of spiral bevel gear with a distinctive feature: their axes do not intersect, unlike typical bevel or spiral bevel gears. Instead of a pitch cone, hypoid gears use a hyperboloidal pitch surface, which allows for a larger offset between the driving and driven shafts. This makes them particularly useful in automotive rear-axle drive systems, as the offset permits the drive shaft to sit lower in the vehicle, effectively lowering the center of gravity.

In hypoid gears, the teeth engage with a sliding motion along their length, similar to how a worm gear operates. This sliding contact, while beneficial for smoothness, leads to greater heat generation than with bevel gears.

For this reason, extreme-pressure (EP) lubricants are essential for proper performance and longevity. These specialized lubricants reduce friction and wear under the high-contact stresses associated with hypoid gear operation.

Another advantage of hypoid gears is the increased strength of the pinion. Due to the nature of the hypoid offset, the pinion gear can have a larger shaft diameter than a similarly rated spiral bevel pinion. This results in improved load-carrying capability and enhanced durability, especially important in high-torque applications like automotive drivetrains.

Related Gear Types

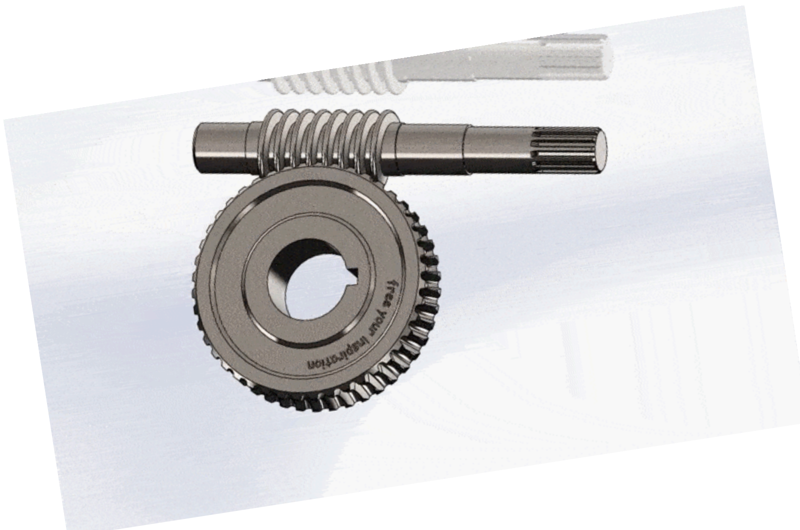
Spiroid gears, which feature a pinion that resembles a conical worm, offer high gear reduction in a compact space and are also used for **high velocity ratios**.

Spiroid gears	Angular gears	Miter gears
		

Angular gears are other variations of bevel-type gears where intersecting angles vary, sometimes exceeding or falling short of the typical 90° arrangement.

Miter gears are a special class of **bevel gears** used to transmit motion between **intersecting shafts at a 90° angle**, where both gears have **the same number of teeth**. This configuration results in a **velocity ratio of 1:1**, meaning both gears rotate at the same speed but with a change in direction of rotation.

Worm gearing is a mechanical arrangement used to transmit power between non intersecting shafts, typically at right angles to one another. The system consists of a worm (which resembles a screw) and a worm gear (a toothed wheel). One of the most notable advantages of worm gears is that they provide high velocity ratios within a compact design, making them suitable for applications where space is limited.



Unlike spur gears where teeth engage with impact, the worm's threads slide smoothly along the gear teeth. This continuous sliding contact contributes to quiet operation, provided that the gear design, manufacturing, and lubrication are properly handled. However, this same sliding contact can also generate significant frictional heat, making overheating a concern under heavy loads or poor lubrication. Therefore, in worm gear design, we must consider not only strength and wear but also heat dissipation.

Gear Ratio and Applications

Worm gears are ideal for achieving high gear ratios in a single stage. Commonly used sets have ratios ranging from 10:1 to 100:1, with some systems going as high as 500:1. For comparison, helical gear systems typically do not exceed 10:1 in a single stage—making worm gears more suitable when significant speed reduction is needed in limited space.

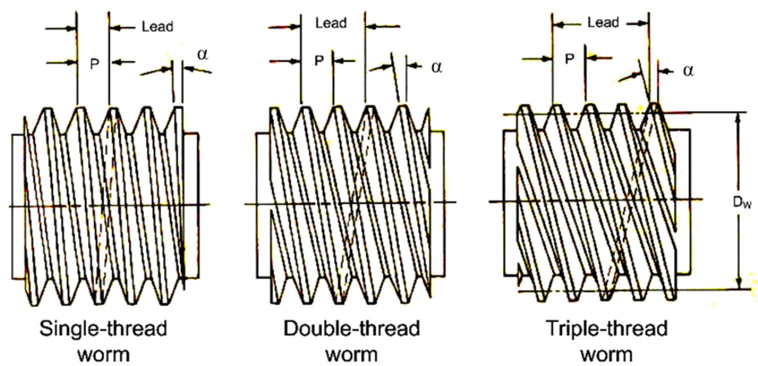
These characteristics make worm gears suitable for:

- Elevators and hoists
- Automotive steering systems
- Conveyor belts
- Tuning instruments (like guitar tuners)

Advantages	Disadvantages
High reduction ratios in a compact space Quiet and smooth operation Self-locking capability in some configurations	Lower mechanical efficiency due to sliding friction Higher heat generation Requires special lubricants (typically EP lubricants)

Lead, L

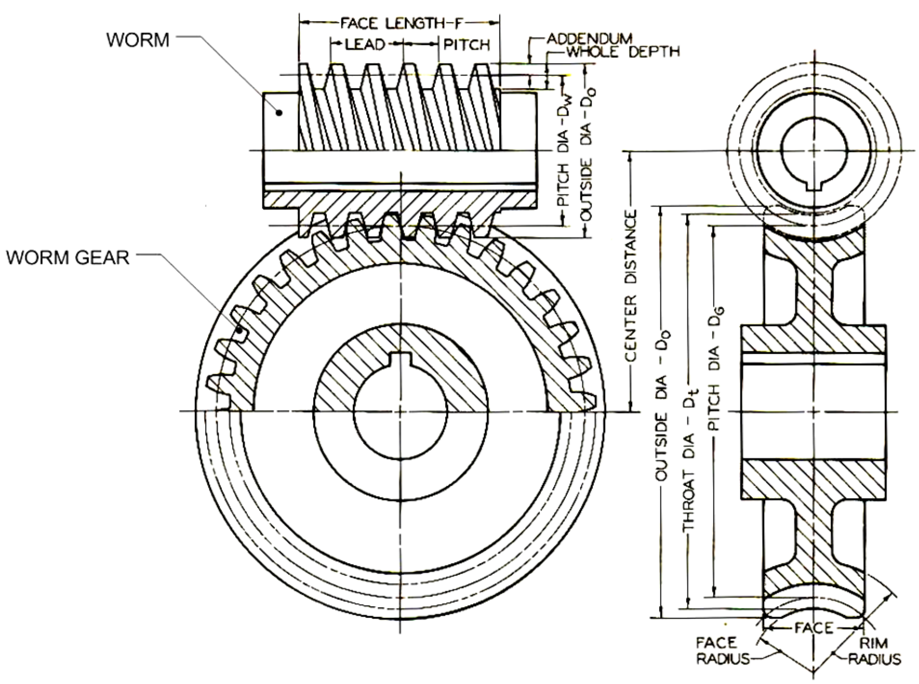
Lead is the distance that the thread advances axially in one turn.



$L = P$ For single thread

$L = 2P$ For double thread

$L = 3P$ For triple thread



Lead angle, α

$$\alpha = tan^{-1}\frac{L}{\pi D_W}$$

Normal pressure angle, θ_n

$$tan\,\theta_n = tan\,\theta\,cos\,\alpha$$

Where: θ = pressure angle

Pitch line velocity, v

$$v = \frac{\pi DN}{60}$$

Where: N = angular speed, rpm

Center distance, c

$$c = \frac{D_W + D_G}{2}$$

The circular pitch, P_C of the worm gear is equal to the axial pitch or the worm:

$$P_C = \frac{\pi D_G}{T_G}$$

Velocity ratio, VR

$$VR = \frac{N_W}{N_G} = \frac{D_G}{D_W} = \frac{T_G}{T_W}$$

Where:
N_G = speed of the worm gear
N_W = speed of the worm
D_G = pitch diameter of the worm gear
D_W = pitch diameter of the worm
T_G = number of teeth on the worm gear
T_W = number of threads on the worm

Number of threads on the worm is equal to 1 for single threaded, 2 for double threaded and 3 for triple threaded worm gear.

EFFICIENCY

Unlike spur or helical gears, tooth dimensions in worm gearing are not fully standardized, which presents unique challenges in design. The worm and its mating gear must be designed together, considering the specifics of involute geometry and the hob cutting process. The hob must match the worm's thread profile precisely, making custom tooling and careful design essential.

A key concern in worm gear design is backlash—the slight clearance between mating teeth. It is typically greater in worm gears than in spur gears and is usually controlled by modifying the thread thickness on the worm. Additionally, designers must consider the differential thermal expansion between materials, especially in applications where temperature fluctuations are expected, as this can influence backlash and performance.

It’s also important to recognize that not all worm gear applications require the same level of precision. For example, in instrumentation or servo systems, where torque is low and accuracy is critical, the design emphasis shifts from load capacity to motion control and minimal backlash.

Efficiency of worm gear, ε:

$$\varepsilon = tan\,\alpha\left[\frac{cos\,\theta_n - f\,tan\,\alpha}{cos\,\theta_n\,tan\,\alpha + f}\right]$$

Where: θ_n = normal pressure angle, 0o for square thread

Separating force between worm and gear, S;

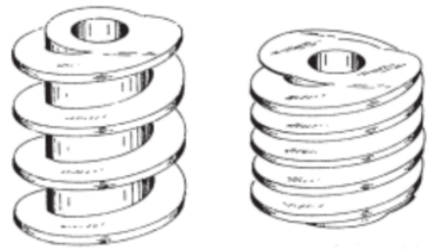
$$S = \frac{F_{tW}\,sin\,\theta_n}{cos\,\theta_n\,sin\,\alpha + f\,cos\,\alpha}$$

$$S = \frac{F_{tG}\,sin\,\theta_n}{cos\,\theta_n\,sin\,\alpha - f\,cos\,\alpha}$$

Where: F_{tG} and F_{tW} are tangential force on the gear and worm, respectively, and can be computed from the transmitted power:

Multi-Threaded Worms

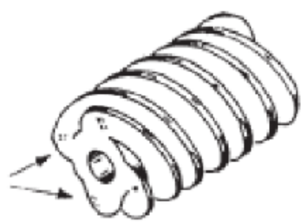
In worm gearing, the term *thread* refers to the helical ridge or “start” wrapped around the cylindrical body of the worm, similar to a screw. A single-threaded worm has one continuous helical path, and with each full rotation of the worm, it advances the worm wheel by one tooth. However, in a double-threaded worm, there are two helical threads that advance side-by-side along the shaft. As a result, one full turn of a double-thread worm will cause the worm wheel to advance by two teeth. This concept significantly influences the gear ratio and efficiency of worm gear systems.



One thread Second thread added.

The number of threads on a worm is also referred to as the number of starts. In industrial applications, worms are commonly manufactured with multiple starts—two, three, four, or even more. The higher the number of starts, the faster the gear engagement and the higher the transmission speed. However, this comes at the cost of reduced mechanical advantage and torque multiplication. A four-start worm, for example, will rotate the worm wheel four teeth per revolution, resulting in a lower reduction ratio but improved output speed and efficiency.

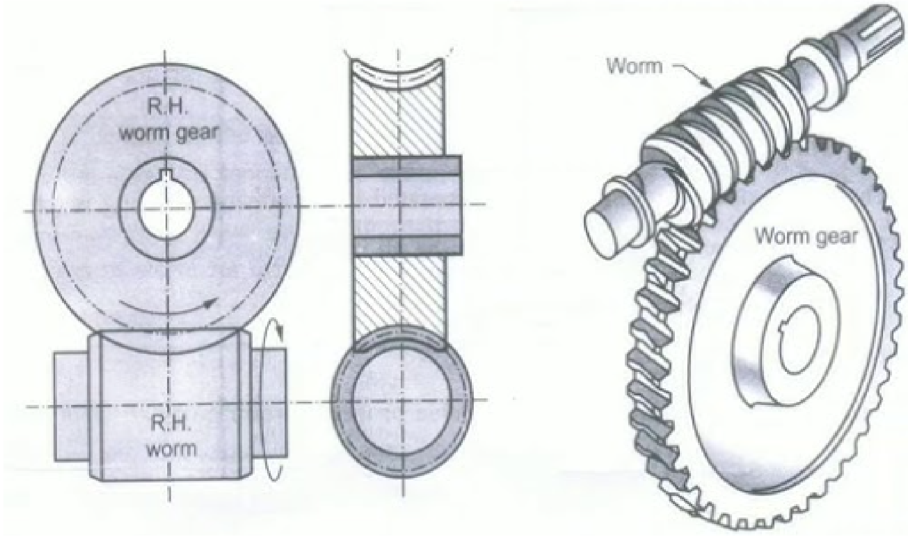
Four starts



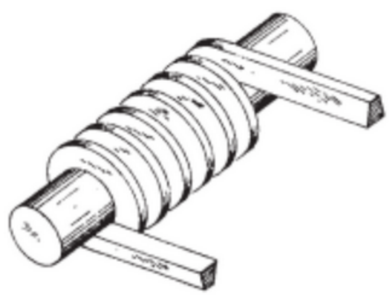
A simple way to identify the number of starts on a worm is by counting the visible thread leads at the end face of the worm shaft. Each lead or start emerges from the shaft and spirals around the cylinder independently. This visual cue is especially useful during inspection or maintenance to confirm the worm type without disassembly.

General Remarks on Worm Gear Design

Designing worm gearing involves special considerations due to the curved, enveloping tooth surfaces and the sliding contact between worm threads and gear teeth. Unlike spur or helical gears that primarily engage with rolling contact, worm gears engage primarily through sliding, leading to complex tooth action and greater sensitivity to friction and heat.

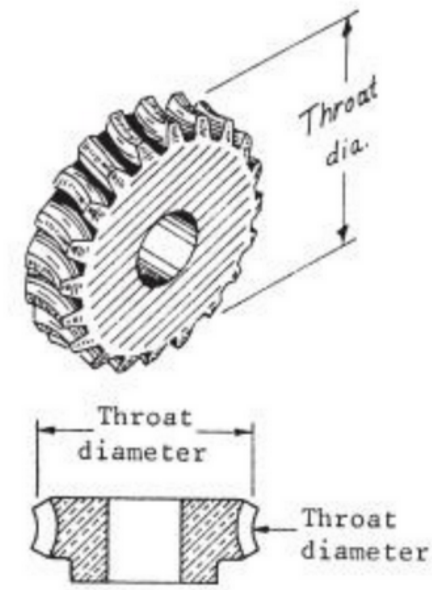


The details of tooth shape and geometry depend on how the threads are cut. Generally, **worm wheels must be hobbed** using a hob that precisely matches the worm's thread form. A list of standard hob sizes is often used as a starting point for design. However, high-precision or custom worm gears may require nonstandard tools to accommodate unique profiles or large lead angles.

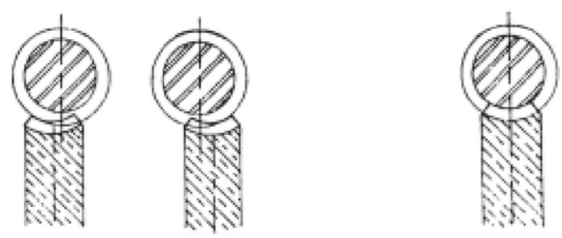


The worm's thread is geometrically similar in form to the tooth profile of a **rack**, which is a straight gear used in rack-and-pinion systems. Just as a rack has straight, evenly spaced teeth designed to engage with a pinion gear, the worm can be visualized as a **helical rack** wrapped around a cylinder. This helical surface is known as the worm thread, and it acts as the driving surface in a worm and worm wheel pair.

One distinctive feature of the worm wheel is the **throat diameter**. The throat diameter refers to the curved, concave surface of the gear that wraps partially around the worm. This curvature allows for improved meshing with the worm threads and provides a larger area of contact, resulting in smoother transmission and higher torque capacity.



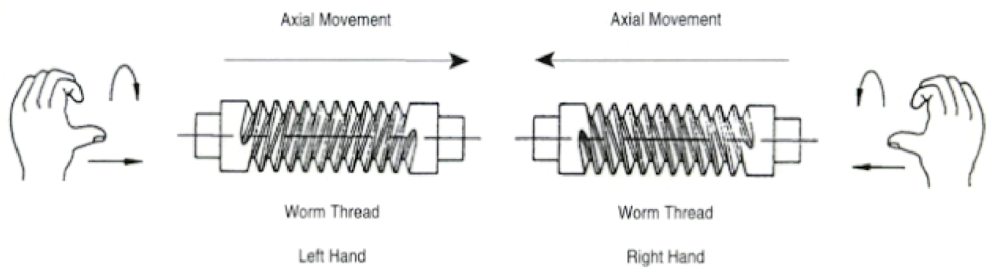
For the worm and worm wheel to operate effectively, it is critical that both elements are properly aligned. The gear teeth on the worm wheel are designed to conform to the helical shape of the worm, meaning any side-to-side misalignment will result in poor tooth contact, increased wear, and reduced efficiency.



As shown in the illustration, correct alignment ensures full surface engagement across the gear face, whereas misalignment leads to partial or incorrect engagement, causing concentrated stress, potential gear failure, and excessive noise.

THREAD HAND

In a **right-hand thread**, which is the most commonly used type in mechanical systems, rotating the worm **clockwise** causes the nut or mating component to move **to the left** along the axis. This follows the conventional tightening motion, similar to turning a bolt with a right-hand thread. Conversely, a **left-hand thread** behaves in the opposite manner; rotating the worm **clockwise** in this case causes axial movement **to the right**. Left-hand threads are used in specific applications where the direction of rotation might otherwise cause a standard thread to loosen.



To easily determine the thread direction, the diagram incorporates the **hand rule**: using your **right hand** for right-hand threads or **left hand** for left-hand threads. When the fingers curl in the direction of rotation, the **thumb points in the direction of axial motion**. This visual technique is helpful for quick identification in field or design scenarios.

LUBRICATION

Worm drives typically have lower efficiency than other gearing types due to the high sliding contact. This is particularly evident at low velocity ratios. While this friction loss can be a disadvantage, it also provides a self-locking feature in certain applications, preventing back-driving.

To combat heat and wear from friction, lubrication is essential. EP (extreme pressure) lubricants are recommended—especially for steel worm and gear bronze combinations. Common lubricants include steam cylinder oil or oils with at least 8–10% acidless tallow, depending on speed and operating temperature. Systems that allow the worm gear to dip into oil (splash lubrication) are generally effective. If cooling is necessary, external return oil systems may be used.

MATERIALS FOR WORM GEARING

The choice of material in worm gear design is critical due to the high sliding contact between the worm and the gear teeth. The **preferred materials for worm wheels** are **bronzes**, specifically **tin bronze** and **nickel-tin bronze**, which provide excellent wear resistance and durability. These may be either chilled cast for a harder surface or unchilled depending on application needs.

For specific applications:

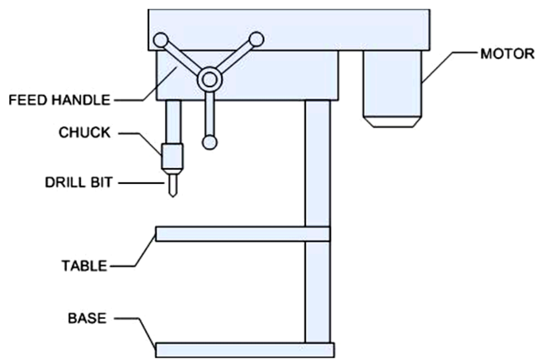
- **Leaded bronze** is ideal for **high-speed** conditions due to its good machinability and running characteristics.
- **Aluminum bronze** and **silicon bronze** are suitable for **heavily loaded, low-speed** applications, providing high strength and thermal conductivity.
- To **reduce cost**, especially in large gears, a **bronze rim** is often mounted on a **cast-iron** or **cast-steel spider** to combine the strength of steel with the wear resistance of bronze.

These material combinations allow designers to balance cost, performance, and manufacturability based on the operating environment.

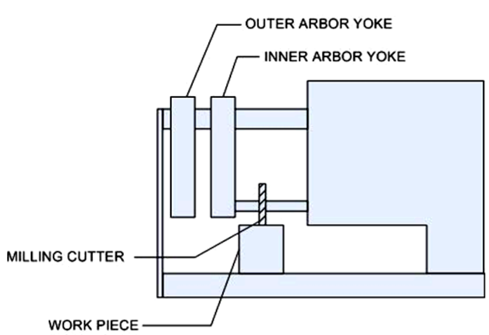
Machine Shop Theory

COMMON MACHINE TOOLS FOUND IN MACHINE SHOPS:

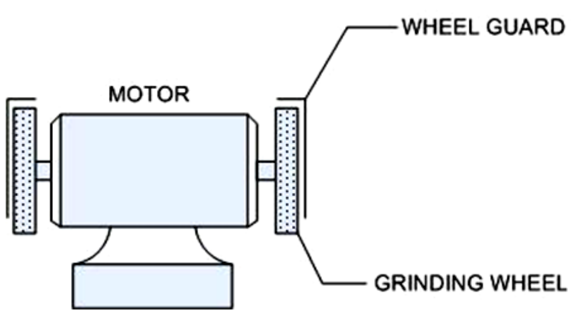
1. Drilling press. Used mainly for producing holes in metal by the use of a cutting tool called drill.



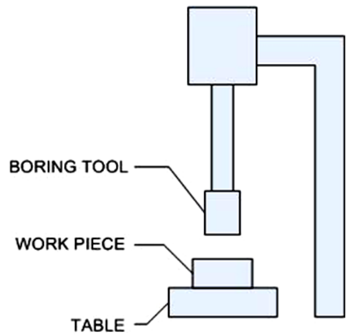
2. Milling machine. Used to form metal parts by removing metal from a workpiece by the use of a revolving cutter with many teeth, each tooth having a cutting edge which removes its share of the stock.



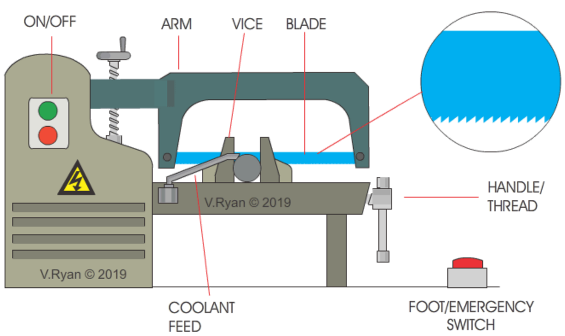
3. Bench grinder. Used to sharpen or shape tools by using an abrasive wheel.



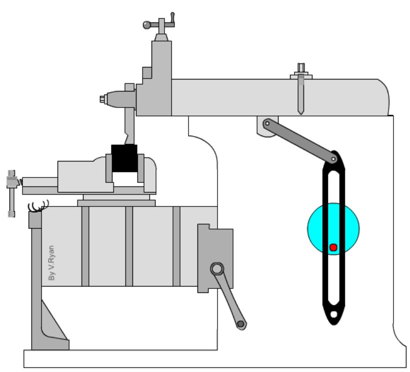
4. Boring machine. Used to enlarge a hole by means of an adjustable cutting tool with only one cutting edge.



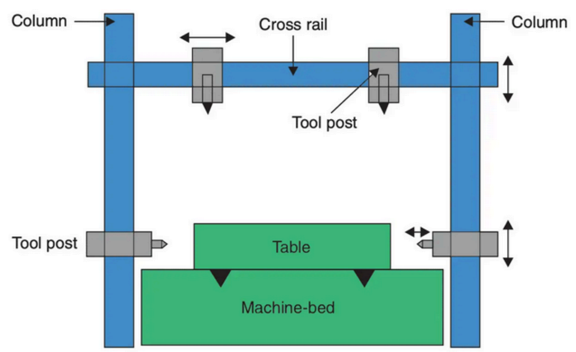
5. Power saw. Used to cut metal of light, medium and large sections using a reciprocating hacksaw blade.



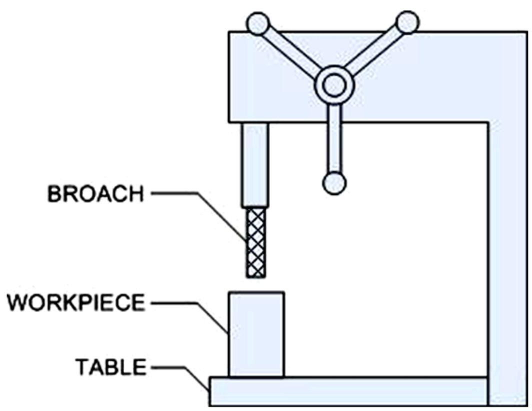
6. Shaper. Used to machine flat or plane surfaces with a single point cutting tool.



7. Planning machine. Used for production of flat surfaces on pieces too large or too heavy to hold in a shaper.



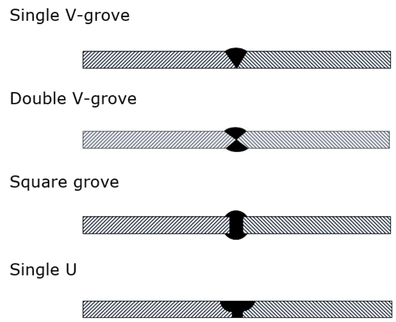
8. Broaching machine. Used to finish internal or external surfaces, such as holes or keyways by the use of a cutter called a broach, which has a series of cutting edges or teeth.



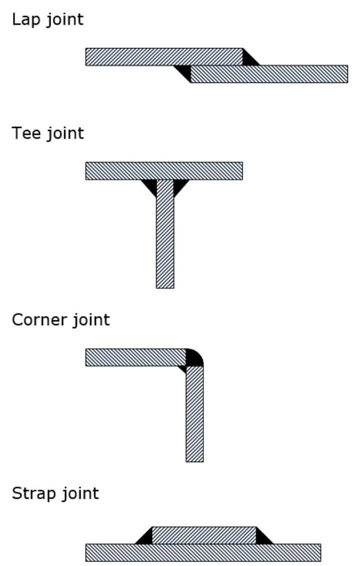
WELDED JOINTS:

Welding is a manufacturing method where parts are joined together by melting the base material and adding filler. It eliminates the need for mechanical fasteners like bolts or rivets and is particularly useful when strength and sealing are crucial. In design, welding must be considered not just from a strength point of view, but also with manufacturing, inspection, and fatigue in mind.

a. Butt weld: These involve joining two plates **end-to-end** in the same plane. The common types are:

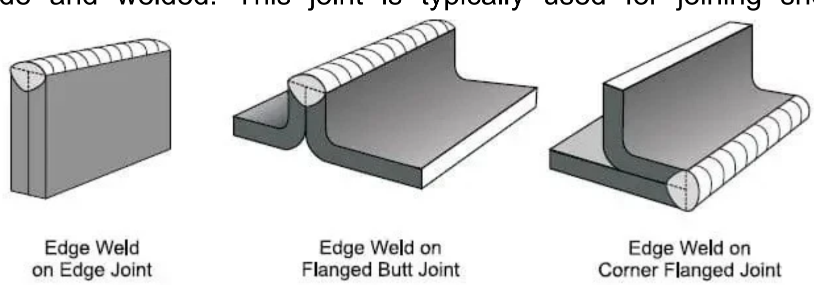


b. Fillet Weld: These are triangular-shaped welds used to join two surfaces at approximately right angles. Common configurations are:

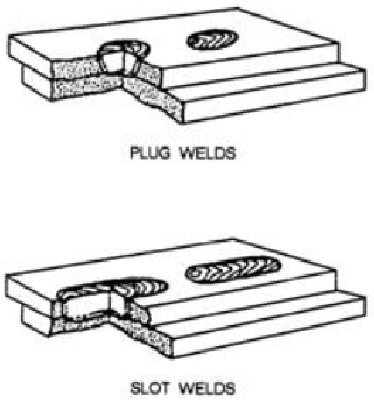


c. Other Weld Types

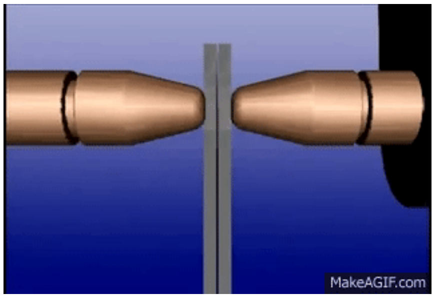
- Edge Joint
  Edge joints are formed when the edges of two metal pieces are placed side by side and welded. This joint is typically used for joining sheet metal parts.



- Plug Weld
  Plug welding involves filling a hole in one metal piece to fuse it to another piece beneath. It's often used as an alternative to spot welding.



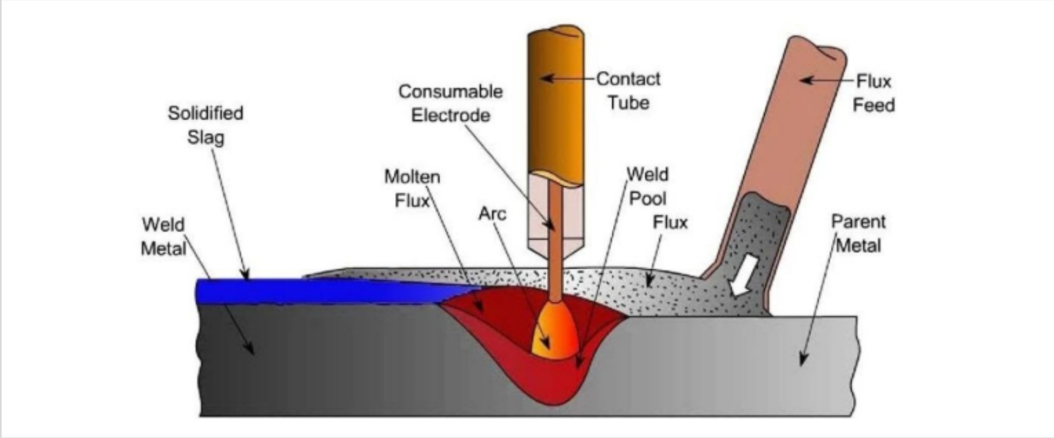
- Spot Weld
  Spot welding is a resistance welding process where metal surfaces are joined by the heat obtained from resistance to electric current flow. It's commonly used for joining sheet metal.



- Intermittent Weld
  Intermittent welding consists of a series of welds spaced at regular intervals. This method reduces distortion and saves welding material.

- Rack Weld
  Rack welding refers to welds used in assembling racks or frameworks, often involving multiple joint types like fillet and groove welds.

- Submerged Arc Welding (SAW)
  SAW is an automated welding process where the arc is submerged under a blanket of granular flux, protecting the weld from atmospheric contamination.

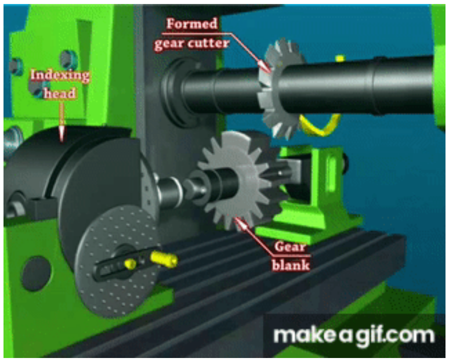


Machining of Gear Teeth

Gears can also be machined by processes like milling, shaping, and hobbing:

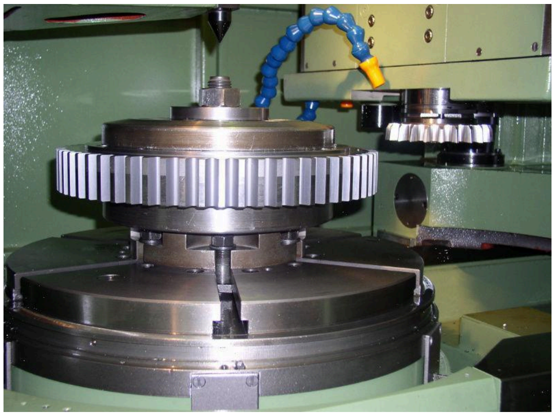
Milling

In gear milling, a form cutter is used to cut individual teeth. While a separate cutter is ideally needed for each gear size due to varying tooth profiles, in practice, a limited number of cutters can cover a range of gear sizes with reasonable accuracy. Milling is often used for prototyping or small-scale production.



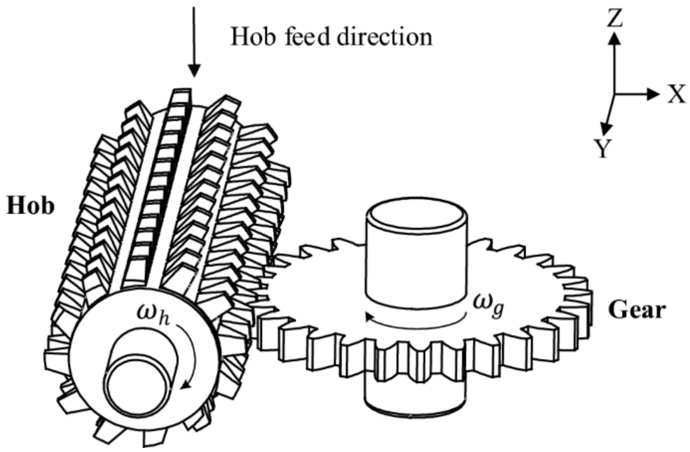
Shaping

Gear shaping involves a reciprocating cutter (pinion or rack cutter) that generates the tooth profile as it moves vertically into the rotating gear blank. This generating process ensures accuracy, particularly for involute profiles, and is effective when internal gears are being cut or when other methods are impractical.



Hobbing

Hobbing is the most popular method for cutting external gears and splines. A hob, which resembles a worm gear, rotates as the gear blank turns. The relative motion between hob and blank generates the gear tooth profile progressively. It is fast, automated, and accurate, making it ideal for mass production. However, it cannot be used to cut internal gears.



Finishing of Gear Teeth

After forming or machining, gears undergo finishing operations to improve surface quality and ensure precise tooth geometry. Finishing is critical for gears operating at high speeds or under heavy loads, where even small errors in tooth profile can cause vibration and noise.

Shaving

A fine finishing process where small amounts of metal are removed from the gear tooth surface, improving the profile and reducing surface roughness.

Burnishing

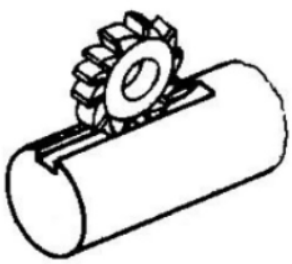
Used after the gear is cut but before heat treatment, this method involves rolling the gear under pressure to smooth the surface without removing material.

Grinding and Lapping

For hardened gears, grinding ensures high accuracy using abrasive wheels that follow the involute profile. Lapping further improves surface finish by sliding the gear teeth against a lapping plate coated with abrasive slurry. This helps distribute the load uniformly across the entire tooth surface.

Illustration:

Calculate the time it will take to mill ¾" by 1.75" in length keyway in a shaft of 2.5" diameter using a 24 tooth cutter turning at 130 rpm and 0.0050" feed per tooth?



Keyway size:
- Width = 0.75 in
- Length = 1.75 in

Shaft diameter = 2.5 in

Cutter:
- Number of teeth = 24
- Speed = 130 rpm
- Feed per tooth = 0.0050 in/tooth

The **cutter type** used is a slitting saw or plain milling cutter. In this setup, the shaft is held horizontally while a horizontal milling cutter, which is a circular blade-like tool, mills the keyway along the axis of the shaft. This method is typically performed using a horizontal milling machine. It is well-suited for creating longer or wider keyways, as it provides high material removal rates and better rigidity during the cutting process.

Calculate the feed rate (in inches per minute)

$$Feed\ rate = (Feed\ per\ tooth)(Number\ of\ teeth)(rpm)$$

$$Feed\ rate = \left(0.0050\frac{in}{tooth}\right)(24\ teeth)(130\ rpm)$$

$$Feed\ rate = 15.6\ \frac{in}{min}$$

Determine the length of cut

We are milling a 1.75" long keyway, so that's the distance the cutter travels linearly.

Calculate time to mill (in minutes)

$$t = \frac{Length\ of\ cut}{Feed\ rate} = \frac{1.75\ in}{15.6\ \frac{in}{min}} = 0.1122\ minutes$$

t = 6.73 sec

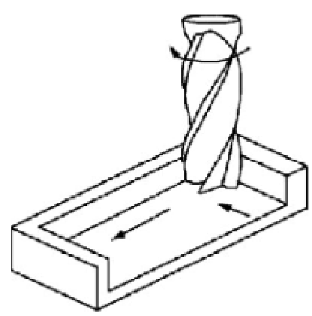
It will take approximately 6.73 seconds to mill the ¾” × 1.75” keyway using the given cutter and parameters.

Illustration:

A roughing work is to be done in a cast steel material in a milling machine. A 1.75-inch diameter cutter is to be used. Find the speed of the cutter, in rpm using:

(a) A high speed steel cutter with 60 fpm cutting speed.

(b) A carbide tipped cutter with 250 fpm cutting speed



End mills and roughing end mills are both used in vertical milling machines for material removal tasks like cutting slots and profiles. They come in various shapes and sizes and can perform roughing or finishing work. Roughing end mills are a specific type of end mill with serrated edges for heavy material removal, but they share the same basic structure and function.

High Speed Steel Cutter: Cutting speed $v = 60\ fpm$

$$v = \pi DN$$

$$\left(60\frac{ft}{min}\right) = \pi(1.75\ in)\left(\frac{1\ ft}{12\ in}\right)N$$

$$N = 131\ rpm$$

Carbide-Tipped Cutter: Cutting speed $v = 250\ fpm$

$$v = \pi DN$$

$$\left(250 \frac{ft}{min}\right) = \pi(1.75\ in)\left(\frac{1\ ft}{12\ in}\right)N$$

$$N = 546\ rpm$$

High Speed Steel (HSS) Cutter – 60 fpm
HSS tools are tough and less brittle, but they have **lower heat resistance**. They lose hardness at high temperatures, so they require **slower cutting speeds** (like 60fps) to prevent overheating and tool wear.

Carbide-Tipped Cutter – 250 fpm
Carbide is much **harder and more heat-resistant** than HSS. It maintains its cutting edge at much higher temperatures, allowing it to cut **faster** (up to 250 fpm or more), which improves productivity in roughing and high-speed machining.

Illustration:

A rectangular steel plate measuring 10 inches in width and 1.75 inches in thickness is to be cut lengthwise using a power hacksaw. The saw operates at 120 strokes per minute, and the average feed per stroke is 0.0070 inches. Calculate the total time required to complete the 10-inch lengthwise cut through the plate.

$$Time = \frac{Length\ of\ cut}{Cutting\ rate}$$

Length of cut = 10 inches (the length being cut through)

Cutting rate = how much material is removed per minute, calculated as

$$Cutting\ rate = \left(120 \frac{strokes}{min}\right)\left(0.0070 \frac{in}{stroke}\right) = 0.84 \frac{in}{min}$$

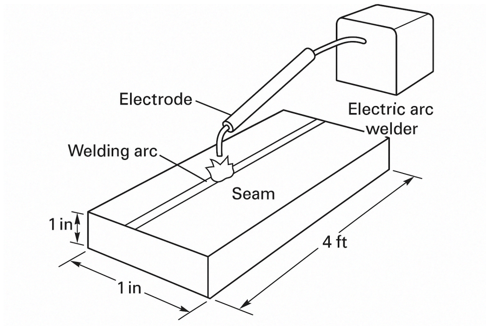
Substitute:

$$Time = \frac{Length\ of\ cut}{Cutting\ rate} = \frac{10\ in}{0.84 \frac{in}{min}}$$

$$Time = 11.9048\ min$$

Illustration:

How long will it take to weld a 1” thick plate by 4 ft long seam using an electric arc weld with a rate of 20 in/min?



$$Time = \frac{Length\ of\ weld}{Welding\ rate} = \frac{48\ in}{20\ \frac{in}{min}}$$

$$Time = 2.4\ minutes$$

Five Heat Treating Practices in Ferrous Metals:

1. Annealing
   Annealing is the process done to increase the ease in machining of metals by heating it above the transformation range (700 to 730 deg C), and afterwards cooling it slowly.

2. Hardening
   Hardening is the process done to increase the hardness of the metal by heating it above transformation range and quenching it usually in oil.

3. Normalizing
   Normalizing is the process done to produce a uniform structure of metals by heating it to 35 - 40 deg C above the transformation range, followed by cooling at room temperature.

4. Stress Relieving
   Stress relieving is the process done to reduce the residual stresses of a metal by heating it to a subcritical temperature (about 590 to 700 deg C) and holding it to that temperature for a suitable time.

5. Tempering
   Tempering is the process done to achieve the desired properties of metals by reheating it to a temperature below the transformation range, followed by any desired rate of cooling.