A flywheel is an energy storage device. In a punching machine application, the power demand is not constant. The actual punching takes only a very small fraction of the machine cycle but requires a huge burst of energy. To prevent the driving motor from stalling or requiring an enormous size, a flywheel is used. The motor runs continuously, storing kinetic energy in the flywheel during the 'idle' part of the cycle. During the actual punching, the flywheel slows down slightly as it releases a portion of this stored energy to perform the work. The design goal is to ensure the flywheel has enough mass so that this speed drop is controlled within a predefined limit, quantified by the coefficient of speed fluctuation C_S.

<u>Step 1:</u> Calculate the Total Punching Energy Required (E_P)

The energy required to punch the hole equals the work done to shear the metal through its entire thickness. We first determine the force needed. The shearing force is calculated as the ultimate shearing stress multiplied by the area being sheared. The shear area (A_S) for a circular hole is the circumference times the thickness.

$$A_S = \pi dt = \pi(0.28\, m)(0.20\, m) = 0.0017593\, m^2$$

The maximum punching force (F_{punch}) is:

$$F_{punch} = S_{Su} A_S = \left(420\, x\, 10^6 \frac{N}{m^2}\right)\left(0.0017593\, m^2\right) = 738{,}903\, N = 738.9\, kN$$

The work done, which is the total punching energy (E_P), can be approximated as half the product of the maximum force and the plate thickness, assuming the force linearly decreases as the punch travels.

$$E_p = \frac{1}{2} F_{punch} t = \frac{1}{2}(738.9\, kN)(0.020\, m) = 7389.03\, J$$

<u>Step 4:</u> Analyze Operating Speeds and Velocity Fluctuation

The problem gives us the maximum speed (N_2 = 220 rpm) and the coefficient of speed fluctuation (C_f = 0.12).

$$C_f = \frac{N_{max} - N_{min}}{N_{avg}}$$

Since the punch is the max load event, the speed drops from maximum ($N_{max} = N_2$) to a minimum ($N_{min} = N_1$) immediately after punching. Let's find N_1 and N_{avg}. Let x = N_{avg}. Then from the definition of average: $N_{max} + N_{min} = 2x$. We can create two equations:

1. $N_{max} - N_{min} = C_S N_{avg}$

   $N_2 - N_1 = 0.12x$

2. $N_2 + N_1 = 2x$

Solving simultaneously:

$$\left(N_2 + N_1\right) - \left(N_2 - N_1\right) = 2x - 0.12x$$

$$2N_2 = 1.88x$$

$$N_1 = 0.94N_{avg}$$

Substituting to (2) we can solve for the following:

$$N_{avg} = 207.547\ rpm$$

<u>Step 5:</u> Relate Rotational Energy Fluctuation to Rim Mass

The total punching energy is supplied by the flywheel. As it performs the work, its kinetic energy drops. This change in kinetic energy is equal to the punching energy. $\Delta KE = E_P$

The fundamental rotational kinetic energy equation, defined in terms of mean rim velocity, is:

$$\Delta KE = mC_f\left(v_{avg}\right)^2 = E_P$$

This equation is derived assuming the rim mass is concentrated at the mean radius, which matches our practical design assumption. The average peripheral velocity (v_{avg}) is found using the mean diameter and average rpm: $v_{avg} = \frac{\pi D_m N_{avg}}{60}$

$$v_{avg} = \frac{\pi\ (1.2\ m)\left(N_{avg}=207.547\ rpm\right)}{60} = 13.0407\frac{m}{s}$$

<u>Step 6:</u> Solve for the Flywheel Mass (m)

We now have all the necessary components for the final equation.

$$7389.03\ J = m(0.12)\left(13.0407\frac{m}{s}\right)^2$$

$$m = 362.083\ kg$$

A flywheel is the mechanical equivalent of a capacitor in an electrical circuit. It stores potential energy in the form of inertia during periods of low demand and releases a sudden 'spike' of energy on demand, preventing system voltage (speed) collapse. The effectiveness of a flywheel scales linearly with mass, but exponentially with radius (r^2). This is why practical engineering designs use a heavy outer rim (at max diameter) and a lightweight web or arms to connect it to the hub]. Every gram of mass placed on the rim provides far more energy storage per kg than mass placed near the hub