Answer Key

Progressions

1

3rd term : 30
9th term : 12

Find 11th term

HP = 30 and 12

$AP = \frac{1}{30}$ and $\frac{1}{12}$

$a_{11} = a_1 + (n-1)d$

From the 3rd term:

$\frac{1}{30} = a_1 + (3-1)d$

$\frac{1}{30} = a_1 + 2d$

$a_1 = \frac{1}{30} - 2d$

From the 9th term

$\frac{1}{12} = a_1 + (9-1)d$

$\frac{1}{12} = a_1 + 8d$

$a_1 = \frac{1}{12} - 8d$

Equating, to solve for a_1 & d ;

$\frac{1}{30} - 2d = \frac{1}{12} - 8d$

$6d = \frac{1}{20}$

$d = \frac{1}{120}$

$a_1 = \frac{1}{60}$

Hence;

$L_{11} = \frac{1}{60} + (11-1)\left(\frac{1}{120}\right)$

$L_{11} = \frac{1}{10}$

$\boxed{HP = 10}$

2

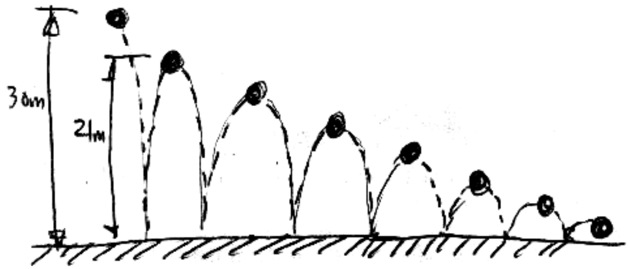
Arithmetic Progression

4, 9, 14, 19 ... up to 22nd term

$$S = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$= \frac{22}{2}\left[2(4) + (22-1)(5)\right]$$

$$\boxed{S = 1243}$$



INFINITE Geometric progression

Given: $h = 30\,m$

$a_1 = 30m(0.70) = 21m$

$a_2 = 21m(0.70) = 14.70\,m$

$r = \dfrac{14.70m}{21\,m} = 0.70$

$D = 30 + 2$ [sum of height of each Bounce]

$D = 30 + 2\left[\dfrac{a_1}{1-r}\right] = 30 + 2\left[\dfrac{21}{1-0.70}\right]$

$\boxed{D = 170\,m}$

4

Given: Harmonic Progression

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$$

$$a_{28} = a_1 + (n-1)d$$

$$= 2 + (28-1)(2)$$

$$a_{28} = 56$$

$$\boxed{HP = \frac{1}{56}}$$

5

Given: Infinite Geometric Series

104, 52, 26, 13

$$S = \frac{a_1}{1-r} = \frac{104}{1-\frac{1}{2}}$$

$$\boxed{S = 208}$$

6

Given: $a_1 = 22\text{ ft}\left(\frac{2}{3}\right) = \frac{44}{3}\text{ ft}$

$$a_2 = \frac{44}{3}\text{ ft}\left(\frac{2}{3}\right) = \frac{88}{9}\text{ ft}$$

$$r = \frac{a_2}{a_1} = \frac{88/9\text{ ft}}{44/3\text{ ft}} = \frac{2}{3}$$

$$S = \frac{a_1}{1-r} = \frac{44/3\text{ ft}}{1 - 2/3}$$

$$\boxed{S = 44\text{ ft}}$$

Given : Infinite Geometric Series

$$S = \frac{a_1}{1-r} = \frac{10}{1-0.5}$$

$$\boxed{S = 20}$$

Given: $a_1 = 25\text{ ft}\left(\frac{5}{7}\right) = \frac{125}{7}\text{ ft}$

$a_2 = \frac{125}{7}\text{ ft}\left(\frac{5}{7}\right) = \frac{625}{49}\text{ ft}$

$$r = \frac{a_2}{a_1} = \frac{(625/49)\text{ ft}}{(125/7)\text{ ft}} = \frac{5}{7}$$

$$a_6 = a_1 r^{n-1}$$

$$= \frac{125}{7}\text{ ft}\left(\frac{5}{7}\right)^{6-1}$$

$a_6 = 3.3203\text{ ft} \longrightarrow$ one bounce up

$a_6 = (3.3203\text{ ft})(2) \longrightarrow$ including bounce down

$$\boxed{a_6 = 6.6405\text{ ft}}$$

9

Given: Geometric Progression

5, 10, 20 . . .

$n = 8$

$r = \frac{10}{5} = 2$

$$S = \frac{a_1(r^n - 1)}{r - 1} = \frac{5(2^8 - 1)}{2 - 1}$$

$$\boxed{S = 1275}$$

10

$a_3 = 12$

$a_6 = 96$

$a_{15} = a_1 r^{n-1}$

$a_3 = a_1 r^{n-1}$

$12 = a_1 r^{3-1}$

$12 = a_1 r^2$

$a_6 = a_1 r^{n-1}$

$96 = a_1 r^{6-1}$

$96 = a_1 r^5$

equating a_1 to solve for r;

$$\frac{12}{r^2} = \frac{96}{r^5}$$

$$r^3 = \frac{96}{12} = 8$$

$r = 2$

$a_1 = 3$

Hence;

$a_{15} = 3(2)^{15-1}$

$$\boxed{a_{15} = 49152}$$

11

Given: Geometric progression

$r = 2$

$n = 8$

$a_8 = 128$

$a_8 = a_1 r^{n-1}$

$128 = a_1 (2)^{8-1}$

$\boxed{a_1 = 1}$

12

Given: Arithmetic progression

$$a_{14} = a_1 + (n-1)d$$
$$= 6 + (14-1)(-5)$$
$$\boxed{a_{14} = -59}$$

13

Given: Arithmetic Mean

$n = 12$

$b = 30$ in (bottom rung)

$a = 18$ in (top rung)

$$S = \left(\frac{a+b}{2}\right)n = \left(\frac{18+30}{2}\right)(12)$$

$$\boxed{S = 288 \text{ in}}$$

Arithmetic Mean $= \dfrac{a+b}{2}$

$$A.M = \frac{12+4}{2}$$

$$\boxed{A.M. = 8}$$

Arithmetic Progression

3rd term = 8

11th term = 80

Find: common Difference, d

$a_3 = a_1 + (n-1)d$

$8 = a_1 + (3-1)d$

$a_1 = 8 - 2d$

$a_{11} = a_1 + (n-1)d$

$80 = a_1 + (11-1)d$

$a_1 = 80 - 10d$

equate a_1 to solve for d

$8 - 2d = 80 - 10d$

$8d = 72$

$\boxed{d = 9}$

16

Geometric Mean

$$GM = \sqrt{(49)(900)}$$

$$\boxed{GM = 210}$$

17

$a = 3$
$d = 1$
$S = 75$

$$S = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$75 = \frac{n}{2}\left[2(3) + (n-1)(1)\right]$$

$$\boxed{n = 10}$$

18

Given: 5th term = 14
8th term = 23

$a_5 = a_1 + (n-1)d$

$14 = a_1 + (5-1)d$

$a_1 = 14 - 4d$

$a_8 = a_1 + (n-1)d$

$23 = a_1 + (8-1)d$

$a_1 = 23 - 7d$

equating a_1 to solve for d

$14 - 4d = 23 - 7d$

$d = 3$

$a_1 = 2$

Hence;

$a_{33} = a_1 + (n-1)d$

$= 2 + (33-1)(3)$

$\boxed{a_{33} = 98}$

Exponential LEAP ONLINE REVIEW

Finite Sequence: 20, 27, 34, 41 ... 69

$a_1 = 20$

$a_n = 69$

$d = 7$

$a_n = a_1 + (n-1)d$

$69 = 20 + (n-1)(7)$

$n = 8$

Hence;

$$S = \left(\frac{a_1 + a_n}{2}\right)(n) = \left(\frac{20+69}{2}\right)(8)$$

$$\boxed{S = 356}$$

20

Given: 6, x, 22

$a_1 = 6$ $n = 3$

$a_3 = 22$ $d = x - 3$

$$a_3 = a_1 + (n-1)d$$

$$22 = 6 + (3-1)(x-3)$$

$$16 = 2(x-3)$$

$$8 = x - 3$$

$$\boxed{x = 11}$$

Given: 8 to 180

$a_1 = 8 \qquad d = 2$

$a_L = 180$

$$a_L = a_1 + (n-1)d$$

$$180 = 8 + (n-1)2$$

$$n = 87$$

$$S = \left(\frac{a_1 + a_L}{2}\right) n$$

$$= \left(\frac{8 + 180}{2}\right)(87)$$

$$\boxed{S = 8178}$$

$S \propto \sqrt{L}$

$S = k\sqrt{L}$

$45\ ft/sec = k\sqrt{30\ ft}$

$k = 8.2158$

Hence;

$S = 8.2158\sqrt{120\ ft}$

$\boxed{S = 90\ ft/sec}$

23

$n = 9$

$a_9 = \frac{73}{51}$

$r = \frac{3}{2}$

Find; First term

$a_9 = a_1 r^{n-1}$

$\frac{73}{51} = a_1 \left(\frac{3}{2}\right)^{9-1}$

$\boxed{a_1 = 0.05850}$

Exponential LEAP ONLINE REVIEW

Given: $n = 25$

$a_{25} = 30$

$d = 4$

$a_{25} = a_1 + (n-1)d$

$30 = a_1 + (25-1)(4)$

$\boxed{a_1 = -66}$

25

Given: $n=6$ $\quad n=16$

$a_6=22 \quad a_{16}=60$

$a_6=a_1+(n-1)d \qquad a_{16}=a_1+(n-1)d$

$22=a_1+(6-1)d \qquad 60=a_1+(16-1)d$

$a_1=22-5d \qquad a_1=60-15d$

equating a_1 to solve for d

$$22-5d=60-15d$$

$$10d=38$$

$$\boxed{d=\frac{19}{5}}$$

Given: 2, 5, 8, 11

$a_1 = 2$

$d = 3$

$n = 23$

$$a_{23} = a_1 + (n-1)d$$

$$= 2 + (23-1)(3)$$

$$\boxed{a_{23} = 68}$$

27

Given: $S = 3$

$a_1 = \frac{1}{4}$

$$S = \frac{a_1}{1 - r}$$

$$3 = \frac{1/4}{1 - r}$$

$$\boxed{r = \frac{11}{12}}$$