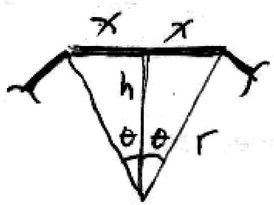
Answer Key

Plane Geometry and Solid Geometry

1

Given: Pentagon $n = 5$

$L = 60 \text{ in}$



$$A = \frac{1}{2} L h n$$

$$P = 2xn$$

$$60\text{in} = 2x(5)$$

$$x = 6$$

$$L = 2x = 12$$

$$\theta = \frac{360^\circ}{2n} = \frac{360^\circ}{2(5)} = 36^\circ$$

$$\tan\theta = \frac{x}{h}$$

$$\tan(36^\circ) = \frac{6}{h}$$

$$h = 8.2583$$

Hence;

$$A = \frac{1}{2}(12)(8.2583)(5)$$

$$\boxed{A = 247.749 \text{ in}^2}$$

2

Given: $D = 9$

$$D = \frac{n}{2}(n-3)$$

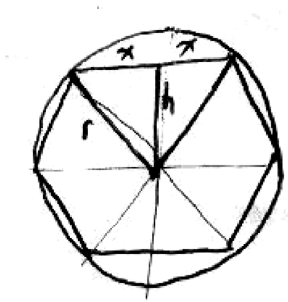
$$9 = \frac{n}{2}(n-3)$$

$$18 = n^2 - 3n$$

$$n^2 - 3n - 18 = 0$$

$\boxed{n = 6}$ (hexagon)

3



$A_c = \pi r^2$

$160 \text{ in}^2 = \pi r^2$

$r = 7.1365 \text{ in}$

$\theta = \frac{360^\circ}{2(6)} = 30^\circ$

$\sin 30^\circ = \frac{x}{7.1365 \text{ in}}$

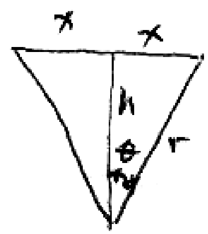
$x = 3.5683 \text{ in}$

Hence;

$L = 2x = 2(3.5683 \text{ in})$

$\boxed{L = 7.1365 \text{ in}}$

4



$$\theta = \frac{360^\circ}{2n} = \frac{360^\circ}{2(5)} = 36^\circ$$

$$\tan 36^\circ = \frac{2.5\text{ in}}{r}$$

$$r = 3.4409\text{ in}$$

Hence;

$$\boxed{d = 6.8819\text{ in}}$$

5

Given:

$n = 18$

$$d = \frac{n}{2}(n-3)$$

$$= \frac{18}{2}(18-3)$$

$$\boxed{d = 135}$$

6

Given: $n=7$

$A_c = 160 cm^2$

req'd: length of side of heptagon, x

$A_c = \pi r^2$

$160 cm^2 = \pi r^2$

$r = 7.1365 cm$

$\theta = \frac{360°}{2n} = \frac{360°}{2(7)}$

$\theta = 25.7143°$

Then;

$\tan\theta = \frac{x}{r}$

$\tan 25.7143° = \frac{x}{7.1365 cm}$

$x = 3.4368 cm$

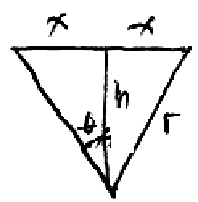
hence;

$L = 2x = 2(3.4368 cm)$

$\boxed{L = 6.8735 cm}$

7

$h = 6\,in$ (apothem)



$L = 2x$

$$\phi = \frac{360^\circ}{2n} = \frac{360^\circ}{2(6)} = 30^\circ$$

$$\tan 30^\circ = \frac{x}{h} = \frac{x}{6\,in}$$

$$x = 2\sqrt{3}$$

Hence;

$$L = 2\left(2\sqrt{3}\right)\,in$$

$$\boxed{L = 6.92\,in}$$

8

Sum of interior angle of nonagon

$$S = (n-2)\,180^\circ$$
$$= (9-2)(180^\circ)$$
$$\boxed{S = 1260^\circ}$$

9

interior angle of decagon

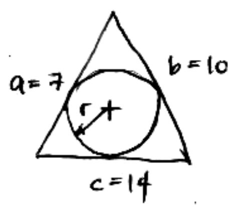
$$\theta = \frac{(n-2)}{n} 180^\circ$$

$$\theta = \frac{(10-2)}{10} 180^\circ$$

$$\boxed{\theta = 144^\circ}$$

10

What is the radius of a circle inscribed in a triangle with sides of 7, 10 and 14-in?



$A = rs$; $s = \dfrac{a+b+c}{2} = \dfrac{7+10+14}{2} = \dfrac{31}{2}$ in

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\frac{31}{2}\left(\frac{31}{2}-7\right)\left(\frac{31}{2}-10\right)\left(\frac{31}{2}-14\right)}$$

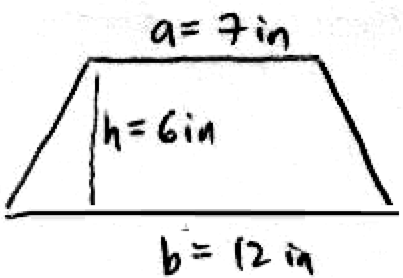
$$A = 32.9687 \text{ in}^2$$

Hence;

$$32.9687 \text{ in}^2 = r\left(\frac{31}{2}\text{in}\right)$$

$$\boxed{r = 2.0605 \text{ in}}$$

11



$$A=\left(\frac{a+b}{2}\right)h=\left(\frac{7in+12in}{2}\right)(6\ in)$$

$$\boxed{A=57\ in^2}$$

12

EQUILATERAL TRIANGLE

$$A_T = 420 \text{ in}^2$$

$$A_T = \frac{\sqrt{3}}{4} x^2$$

$$420 \text{ in}^2 = \frac{\sqrt{3}}{4} x^2$$

$$\boxed{x = 31.1440 \text{ in}}$$

length of side of Equilateral triangle

13

Rectangle: $W = 12m$

$A_R = 100 m^2$

$P_R = P_T$

$P_R = 2L + 2W$

$A_R = LW$

$100m^2 = L(12m)$

$L = 8.3333 \ m$

$P_R = 2(8.3333m) + 2(12m)$

$P_R = 40.6667 \ m$

and;

$P_T = a + b + c = 3x$ (for equilateral triangle)

equating;

$40.6667m = 3X$

$x = 13.5556 \ m$ length of side of equilateral triangle

Hence;

$$A_T = \frac{\sqrt{3}}{4} x^2 = \frac{\sqrt{3}}{4}(13.5556 \ m)^2$$

$$\boxed{A_T = 79.5674 \ m^2}$$

14

The area of a circle is 310-in². What is the circumference?

$A_c = 310 \text{ in}^2$

Find: circumference, c

$C = 2\pi r$; $A_c = \pi r^2$

$310 \text{ in}^2 = \pi r^2$

$r = 9.9336 \text{ in}$

Hence;

$C = 2\pi(9.9336 \text{ in})$

$\boxed{C = 62.4145 \text{ in}}$

15

$A_c = 260 \text{ in}^2$

Find: diameter, d

$$A_c = \frac{\pi d^2}{4}$$

$$260 \text{ in}^2 = \frac{\pi d^2}{4}$$

$$\boxed{d = 18.1946 \text{ in}}$$

16

Ellipse

Aspect ratio = 2.5

P = 28 in

$A = \pi ab$; $A.R. = \frac{a}{b}$

$2.5 = \frac{a}{b}$

$a = 2.5b$

and;

$P = 2\pi\sqrt{\frac{a^2+b^2}{2}}$

$28\text{ in} = 2\pi\sqrt{\frac{(2.5b)^2+b^2}{2}}$

$b = 2.3406\text{ in}$

$a = 5.8515\text{ in}$

Hence;

$A = \pi(5.8515\text{ in})(2.3406\text{ in})$

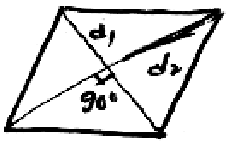
$\boxed{A = 43.0273\text{ in}^2}$

17

Rhombus:

$d_1 = 34$ in

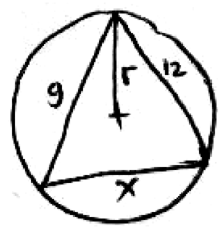
$d_2 = 25$ in



$$A = \frac{d_1 d_2}{2} = \frac{(34 \text{ in})(25 \text{ in})}{2}$$

$$\boxed{A = 425 \text{ in}^2}$$

18



$$A_T = 40\ \text{cm}^2$$

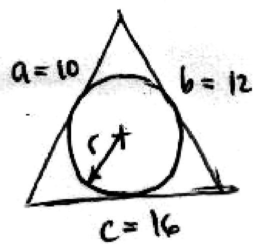
$$r = 8.15\ \text{cm}$$

$$A = \frac{abc}{4r}$$

$$40\ \text{cm}^2 = \frac{(9\ \text{cm})(12\ \text{cm})\,x}{4(8.15\ \text{cm})}$$

$$\boxed{x = 12.0741\ \text{cm}}$$

19



$A = rs$

$$s = \frac{a+b+c}{2} = \frac{10+12+16}{2} = 19\,cm$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{19(19-10)(19-12)(19-16)}$$

$A = 59.9250\ cm^2$

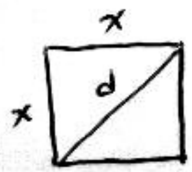
Hence;

$59.9250\,cm^2 = r(19\,cm)$

$$\boxed{r = 3.1540\ cm}$$

20

Given: square

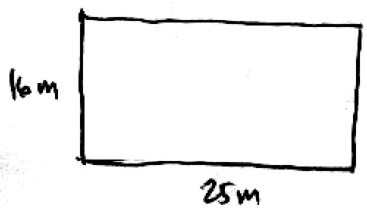


$$d = \sqrt{2}\,x$$

$$18 \text{ in} = \sqrt{2}\,x$$

$$\boxed{x = 12.7279 \text{ in}}$$

21



$$A_r = LW = (25m)(16m) = 400\ m^2$$

$$A_c = 2A_r = 2(400\ m^2) = 800\ m^2$$

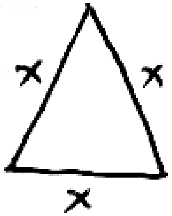
Hence;

$$A_c = \frac{\pi}{4} d^2$$

$$800\ m^2 = \frac{\pi}{4} d^2$$

$$\boxed{d = 31.9154\ m}$$

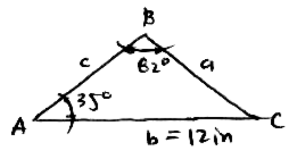
22



$$A = \frac{\sqrt{3}}{4} x^2 = \frac{\sqrt{3}}{4}(12\,\text{in})^2$$

$$\boxed{A = 62.3538\ \text{in}^2}$$

23



$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 35^\circ} = \frac{12\text{ in}}{\sin 82^\circ}$$

$$a = 6.9506 \text{ in}$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$12^2 = 6.9506^2 + c^2 - 2(6.9506)\cos 82^\circ$$

$$c = 9.8805 \text{ in}$$

and;

$$S = \frac{a+b+c}{2} = \frac{(6.9506 + 12 + 9.8805)\text{in}}{2}$$

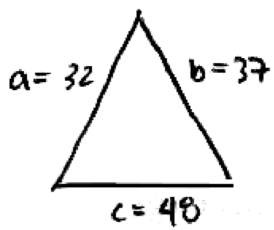
$$S = 14.4156 \text{ in}$$

Hence;

$$A = \sqrt{14.4156(14.4156-6.9506)(14.4156-12)(14.4156-9.8805)}$$

$$\boxed{A = 34.3350 \text{ in}^2}$$

24



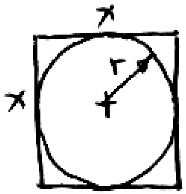
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$S = \frac{a+b+c}{2} = \frac{32+37+48}{2} = 58.5 \text{ in}$$

$$A = \sqrt{58.5(58.5-32)(58.5-37)(58.5-48)}$$

$$\boxed{A = 591.5817 \text{ cm}^2}$$

25



$A_s = 250\ in^2$

$A_s = x^2$

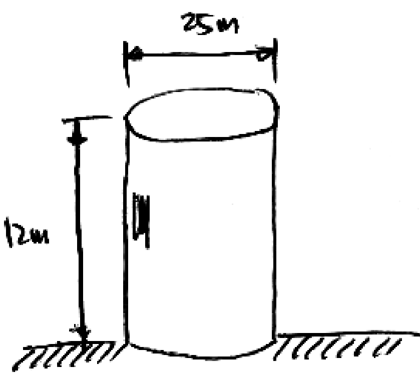
$250\ in^2 = x^2$

$x = 15.8114\ in = d$

$$A_c = \frac{\pi d^2}{4} = \frac{\pi (15.8114\ in)^2}{4}$$

$$\boxed{A_c = 196.3495\ in^2}$$

26



$$A = \pi r^2 + 2\pi r h$$

$$= \pi(12.5m)^2 + 2\pi(12.5m)(12m)$$

$$= 1433.3516 \text{ m}2$$

Hence;

$$\frac{1L}{12m^2} = \frac{x}{1433.3516 \text{ m}^2}$$

$$x = 119.4459$$

$$\boxed{x = 119.5 \text{ L}}$$

27

Given:

$P = 60 in$

$A_s = A_c$

req'd: circumference of circle

$P = 4x$

$60 in = 4x$

$x = 15 in$

$A_s = x^2 = (15 in)^2$

$A_s = 225 in^2$

Equating:

$A_s = A_c$

$225 in^2 = \pi r^2$

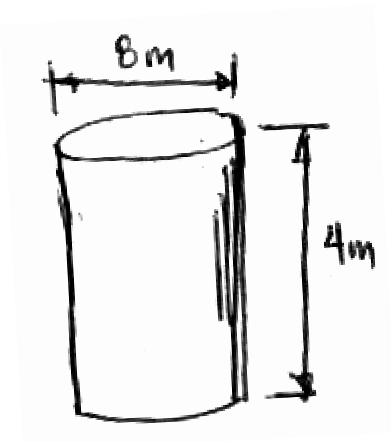
$r = 8.4628 in$

Hence;

$C = 2\pi r = 2\pi(8.4628 in)$

$\boxed{C = 53.1733 in}$

28



surface area, SA

$$SA = 2\pi r^2 + 2\pi r h$$

$$= 2\pi(4m)^2 + 2\pi(4m)(4m)$$

$$\boxed{SA = 201.0619\ m^2}$$

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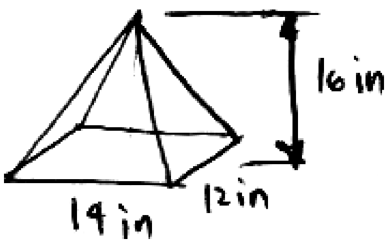
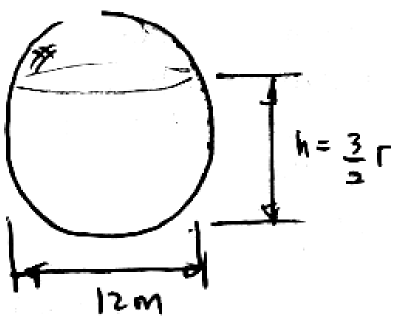


$$V = \frac{1}{3} A h$$

$$= \frac{1}{3}(14\text{ in})(12\text{ in})(16\text{ in})$$

$$\boxed{V = 896\text{ in}^3}$$

30



$$V = \frac{\pi h^2}{3}(3r - h)$$

$$h = \frac{3}{2} r = \frac{3}{2}(6m) = 9m$$

$$V = \frac{\pi (9m)^2}{3}\left[3(6m) - 9m\right]$$

$$\boxed{V = 763.4070 \ m^3}$$

31

Given: Steel plate: 1.5 in × 5 ft × 12 ft

$$\rho_s = 0.285 \frac{lb}{in^3}$$

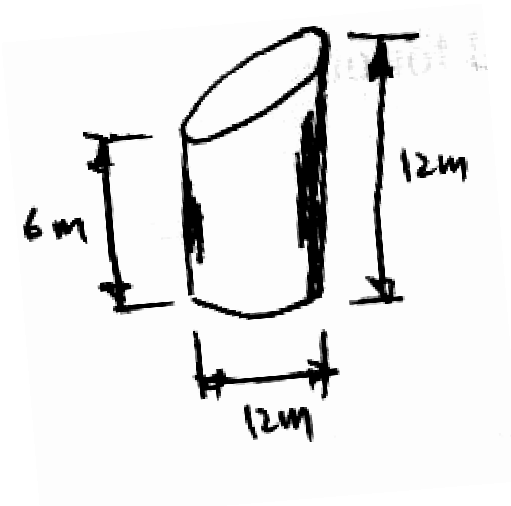
Req'd: weight of the material

$$m = \rho V$$

$$= \left(0.285 \frac{lb}{in^3}\right)\left(\frac{12\,in}{1\,ft}\right)^3 (1.5\,in)\left(\frac{1\,ft}{12\,in}\right)(5\,ft)(12\,ft)$$

$$\boxed{m = 3693.6\ lbs}$$

32



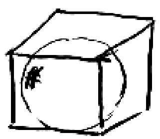
$$V = A\left(\frac{h_1 + h_2}{2}\right)$$

$$= \frac{\pi d^2}{4}\left(\frac{h_1 + h_2}{2}\right)$$

$$= \frac{\pi (12m)^2}{4}\left(\frac{12m + 6m}{2}\right)$$

$$\boxed{V = 1017.8760 \text{ m}^3}$$

33



$d = 6\,ft$

req'd: Diameter of sphere

$$V_c = x^3 ; \quad d = \sqrt{3}\,x \text{ (space diagonal)}$$

$$6\,ft = \sqrt{3}\,x$$

$$x = 3.4641\,ft$$

$$V_c = (3.4641\,ft)^3$$

$$V_c = 41.5692\,ft^3$$

and;

$$V_s = 3V_c = 3(41.5692\,ft^3)$$

$$V_s = 124.7077\,ft^3$$

Hence;

$$V_s = \frac{4}{3}\pi r^3$$

$$124.7077\,ft^3 = \frac{4}{3}\pi r^3$$

$$r = 3.0993\,ft$$

$$\boxed{D = 2r = 6.1987\,ft}$$

34

Sphere: $d_1 = 5\text{ cm}$ $\quad d_2 = 7\text{ cm}$

$r_1 = 2.5\text{ cm}$ $\quad r_2 = 3.5\text{ cm}$

Find: percent increase in surface area.

$SA_1 = 4\pi r_1^2$ $\quad SA_2 = 4\pi r_2^2$

$= 4\pi(2.5\text{ cm})^2$ $\quad = 4\pi(3.5\text{ cm})^2$

$SA_1 = 25\pi\text{ cm}^2$ $\quad SA_2 = 49\pi\text{ cm}^2$

$$\%\text{ increase} = \frac{(49\pi - 25\pi)\text{ cm}^2}{25\pi\text{ cm}^2} \times 100\%$$

$$\boxed{\%\text{ increase} = 96\%}$$

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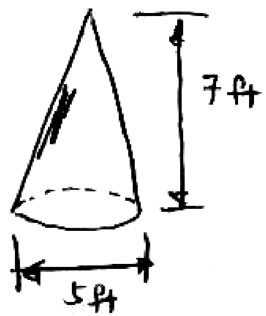

$$SA = \pi r^2 + \pi r\sqrt{r^2 + h^2}$$

$$= \pi(2.5)^2 + \pi(2.5)\sqrt{2.5^2 + 7^2}$$

$$\boxed{SA = 78.0139\ ft^2}$$