

Answer Key

Analytic Geometry-Lines

$$m = -7$$

$$a = 5 \text{ (x-intercept)}$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -7(x - 5)$$

$$y = -7x + 35$$

$$7x - y - 35 = 0$$

$$b = -5 \quad (\text{y-intercept})$$

and parallel to line $6x - 3y = 4$

$$6x - 3y = 4$$

$$[3y = 6x - 4] \frac{1}{3}$$

$$y = 2x - \frac{4}{3}$$

$$m = 2$$

for parallel lines $m_1 = m_2$

$$y = mx + b$$

$$y = 2x + (-5)$$

$$y = 2x - 5$$

$$A(4,5) \text{ and } B(-1,2)$$

$$d_1 = d_2$$

$$\sqrt{(x-x_1)^2 + (y-y_1)^2} = \sqrt{(x-x_2)^2 + (y-y_2)^2}$$

$$(x-4)^2 + (y-5)^2 = (x+1)^2 + (y-2)^2$$

$$(x^2 - 8x + 16) + (y^2 - 10y + 25) = (x^2 + 2x + 1) + (y^2 - 4y + 4)$$

$$\cancel{x^2} - 8x + 16 + \cancel{y^2} - 10y + 25 = \cancel{x^2} + 2x + 1 + \cancel{y^2} - 4y + 4$$

$$-8x - 10y + 41 = 2x - 4y + 5$$

$$[-10x - 6y + 36 = 0] \cdot \frac{1}{2}$$

$$5x + 3y - 18 = 0$$

Distance Between Two parallel lines

$$5x - 4y + 11 = 0$$

$$5x - 4y + 3 = 0$$

$$d = \left| \frac{C_2 - C_1}{\sqrt{A^2 + B^2}} \right| = \left| \frac{11 - 3}{\sqrt{5^2 + (-4)^2}} \right|$$

$$d = 1.2494$$

equation of line: $4x + 5y = 6$

point: $(1, 5)$

Distance from point to a line

$$d = \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}$$

$$= \frac{4(1) + 5(5) + (-6)}{\sqrt{4^2 + 5^2}}$$

$$d = 3.5920$$

$$y = \frac{1}{2}x + 8$$

$$m_2 = -\frac{1}{m_1} \quad \text{lines perpendicular to each other}$$

$$m_2 = -\frac{1}{\frac{1}{2}}$$

$$m_2 = -2$$

Hence,

$$y = -2x + 8$$

$$2x + y - 8 = 0$$

$$4x + 5y = 15$$

$$y = mx + b$$

$$[5y = -4x + 15] \frac{1}{5}$$

$$y = -\frac{4}{5}x + 3$$

Hence;

$$y\text{-intercept} = 3$$

$$m = 2$$

$$P(3, 4)$$

Trial and error:

$$x - y = -1$$

$$3 - 4 = -1$$

$$-1 = -1$$

$$b = -4$$

$$P = (6, 3)$$

$$y = mx + b$$

$$3 = m(6) - 4$$

$$m = \frac{7}{6}$$

$$y = 3x + 3 ; m_1 = 3$$

$$y = 4x + 8 ; m_2 = 4$$

$$\tan \theta = \frac{m_2 - m_1}{1 + m_2 m_1} = \frac{4 - 3}{1 + (4)(3)} = 0.07692$$

$$\theta = 4.3987^\circ$$

$$b = 3$$

$$a = 4$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\left[\frac{x}{4} + \frac{y}{3} = 1 \right] (4)(3)$$

$$3x + 4y = 12$$

$$y - 6x + 3 = 0$$

$$y = 6x - 3$$

$$m = 6$$

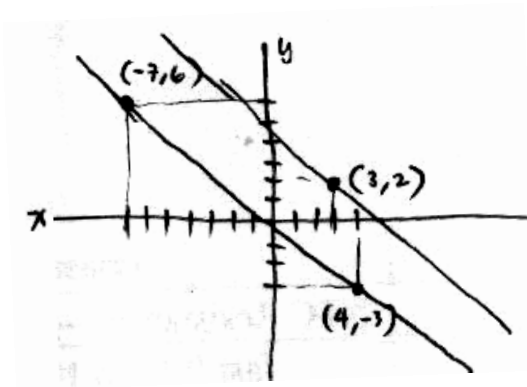
$$P(-7, 5)$$

trial and error:

$$2x + 7y = 21$$

$$2(-7) + 7(5) = 21$$

$$21 = 21$$



trial and error

$$y = \frac{9x}{8} - \frac{11}{8}$$

$$y = \frac{9(3)}{8} - \frac{11}{8}$$

$$y = 2$$

Hence;

$$(3, 2)$$

$$P_1(-7, -4.3)$$

$$P_2(6.3, -8.1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-8.1 + 4.3)}{(6.3 + 7)} = -\frac{2}{7}$$

$$y - y_1 = m(x - x_1)$$

$$y + 8.1 = -\frac{2}{7}(x - 6.3)$$

$$y + 8.1 = -\frac{2}{7}x + \frac{9}{5}$$

$$y = -\frac{2}{7}x - \frac{63}{10}$$

$$P_1(2, 3)$$

$$P_2(9, 10)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 3}{9 - 2} = 1$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 1(x - 2)$$

$$y = x + 1$$

$$\tan \theta = \frac{m_2 - m_1}{1 + m_2 m_1} = \frac{0.9 - 0}{1 + (0.9)(0)} = 0.9$$

$$\theta = 41.9872^\circ$$

Distance between two points

$$P_1(0, 6)$$

$$P_2(7, -2)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(7 - 0)^2 + (-2 - 6)^2}$$

$$d = 10.6301$$

midpoint ; $P_1(3, 6)$

$P_2(-5, -6)$

$$y = \frac{y_2 + y_1}{2} = \frac{-6 + 6}{2} = 0$$

$$x = \frac{x_2 + x_1}{2} = \frac{-5 + 3}{2} = -1$$

midpoint $(-1, 0)$

$$\text{Line 1: } 3x - 5y = 11$$

$$\text{Line 2: } 3x - 5y = 30$$

$$d = \frac{C_2 - C_1}{\sqrt{A^2 + B^2}} = \frac{-3 + 11}{\sqrt{3^2 + (-5)^2}}$$

$$d = 1.3720$$

$$P(1, 3)$$

$$4x - 3y + 5 = 0$$

$$\left[3y = 4x + 5 \right] \frac{1}{3}$$

$$y = \frac{4}{3}x + \frac{5}{3}$$

$$m_1 = \frac{4}{3} = m_2$$

$$y - y_1 = m_2(x - x_2)$$

$$y - 3 = \frac{4}{3}(x - 1)$$

$$\left[y - 3 = \frac{4}{3}x - \frac{4}{3} \right] 3$$

$$3y - 9 = 4x - 4$$

$$4x - 3y + 5 = 0$$

$$a = 3$$

$$b = 8$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\left[\frac{x}{3} + \frac{y}{8} = 1 \right] (3)(8)$$

$$8x + 3y = 24$$

$$b = -9$$

$$m = \frac{5}{4}$$

$$y = mx + b$$

$$\left[y = \frac{5}{4}x - 9 \right] 4$$

$$4y = 5x - 36$$

$$5x - 4y - 36 = 0$$

$$y = 6 - 4t$$

$$x = 3 + t$$

$$t = x - 3$$

Substi:

$$y = 6 - 4(x - 3)$$

$$y = 6 - 4x + 12$$

$$y = -4x + 18$$

$$m = -4$$

$$4x + 3y - 8 = 0 \text{ perpendicular to } 3x - Ay + 3 = 0$$

$$[3y = -4x + 8] \frac{1}{3}$$

$$y = -\frac{4}{3}x + \frac{8}{3}$$

$$m_1 = -\frac{4}{3}$$

$$m_2 = -\frac{1}{m_1} = -\frac{1}{(-4/3)} = \frac{3}{4}$$

from;

$$3x - Ay + 3 = 0$$

$$Ay = 3x + 3$$

$$y = \frac{3}{A}x + \frac{3}{A}$$

$$m = \frac{3}{A} = \frac{3}{4}$$

Hence; $A = 4$

$$P_1(2, -4)$$

$$P_2(-5, 3)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 + 4}{-5 - 2} = -1$$

and;

$$y - y_1 = m(x - x_1)$$

$$y + 4 = -1(x - 2)$$

$$y + 4 = -x + 2$$

$$y = -x - 2$$

$$P_1(-3, 1)$$

$$P_2(5, 0)$$

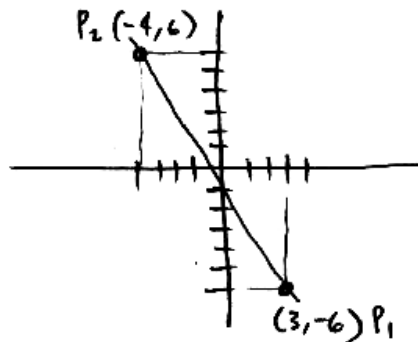
$$P_3(4, 4)$$

$$A = \frac{1}{2} \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

$$A = \frac{1}{2} \begin{bmatrix} -3 & 1 & 1 \\ 5 & 0 & 1 \\ 4 & 4 & 1 \end{bmatrix}$$

$$A = \frac{1}{2} (31)$$

$$A = 15.5 \text{ sq. units}$$



$$x = \frac{x_1 r_2 + x_2 r_1}{r_2 + r_1} ; y = \frac{y_1 r_2 + y_2 r_1}{r_2 + r_1}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-4 - 3)^2 + (6 - (-6))^2}$$

$$d = 13.8924$$

$$r_1 = 13.8924 \left(\frac{3}{6} \right) = 8.3355$$

$$r_2 = 13.8924 - 8.3355 = 5.5570$$

Substi

$$x = \frac{(3)(5.5570) + (-4)(8.3355)}{5.5570 + 8.3355}$$

$$x = -6/5$$

$$y = \frac{(-6)(5.5570) + (6)(8.3355)}{5.5570 + 8.3355}$$

$$y = \frac{6}{5}$$

Hence;

$$\boxed{\left(-\frac{6}{5}, \frac{6}{5}\right)}$$