Answer Key

Conic Sections and Polar Coordinates

1

$P(-5,4)$

$C(5,4)$

$$(x-h)^2+(y-k)^2=r^2$$

$$(-5-5)^2+(4-4)^2=r^2$$

$$100=r^2$$

Hence;

$$\boxed{(x-5)^2+(y-4)^2=100}$$

2

exponential LEAP ONLINE REVIEW

$P(4,9)$

$$x^2+y^2+4x+6y=14$$

$$(x^2+4x+4)+(y^2+6y+9)=14+4+9$$

$$(x+2)^2+(y+3)^2=27$$

$$c(-2,-3)$$

and;

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{(4+2)^2+(9+3)^2}$$

$$\boxed{d=13.4164}$$

3

$$x^2 + y^2 - 3x - 5y - 35 = 0$$

$$x^2 - 3x + \frac{9}{4} + y^2 - 5y + \frac{25}{4} = 35 + \frac{9}{4} + \frac{25}{4}$$

$$\left(x - \frac{3}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = 43.5$$

$$\boxed{c\left(\frac{3}{2}, \frac{5}{2}\right)}$$

4

$$x^2+y^2-7y=0$$

$$x^2+y^2-7y+\frac{49}{4}=\frac{49}{4}$$

$$x^2+\left(y-\frac{7}{2}\right)^2=\frac{49}{4}$$

$$C_1\left(0,\frac{7}{2}\right)$$

$$x^2-6x+y^2-6y-14=0$$

$$x^2-6x+9+y^2-6y+9=14+9+9$$

$$(x-3)^2+(y-3)^2=32$$

$$C_2(3,3)$$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{(3-0)^2+\left(3-\frac{7}{2}\right)^2}$$

$$\boxed{d=3.0414}$$

5

$$x^2 - 8x + y^2 - 6y - 15 = 0$$

$$x^2 - 8x + 16 + y^2 - 6y + 9 = 15 + 16 + 9$$

$$(x-4)^2 + (y-3)^2 = 40$$

$$r^2 = 40$$

$$\boxed{r = 6.3246}$$

6

$$x^2 + y^2 - 6x + 3y = 10$$

$$x^2 - 6x + 9 + y^2 + 3y + \frac{9}{4} = 10 + 9 + \frac{9}{4}$$

$$(x - 3)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{85}{4}$$

$$\boxed{C\left(3, -\frac{3}{2}\right)}$$

7

$$x^2 - 3x + y^2 + 3y = 3$$

$$x^2 - 3x + \frac{9}{4} + y^2 + 3x + \frac{9}{4} = 3 + \frac{9}{4} + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = 7.5$$

$\boxed{c\left(\frac{3}{2}, -\frac{3}{2}\right)}$ and $\boxed{r = 2.7386}$

STANDARD EQUATION of a circle

$c(2,3)$

$P(6,5)$

$$(x-h)^2+(y-k)^2=r^2$$

$$(6-2)^2+(5-3)^2=r^2$$

$$r^2=20$$

and;

$$(x-2)^2+(y-3)^2=20$$

$$x^2-4x+4+y^2-6y+9=20$$

$$\boxed{x^2+y^2-4x-6y-7=0}$$

9

$$x^2 - 3x + y^2 - 4y - 6 = 0$$

$$x^2 - 3y + \frac{9}{4} + y^2 - 4y + 4 = 6 + \frac{9}{4} + 4$$

$$\left(x - \frac{3}{2}\right)^2 + (y - 2)^2 = 12.25$$

$$r^2 = 12.25$$

$$\boxed{r = 3.5}$$

10

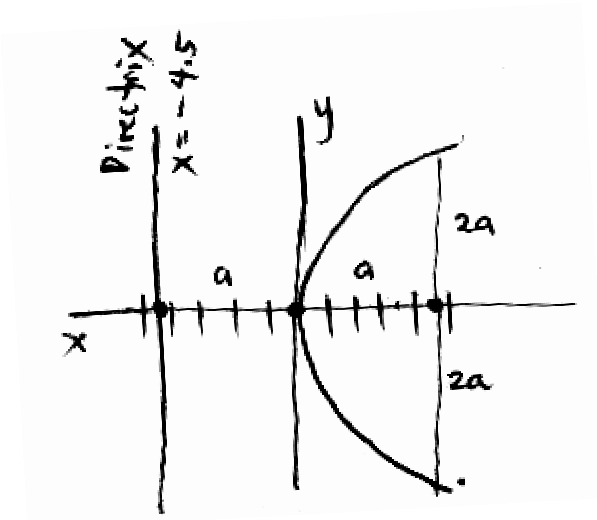
$$\frac{x^2}{49} + \frac{y^2}{25} = 1$$

$$\frac{x^2}{7^2} + \frac{y^2}{5^2} = 1$$

$$LR = \frac{2b^2}{a} = \frac{2(5)^2}{7}$$

$$\boxed{LR = 7.1429}$$

11



$y^2 = 18x$

$y^2 = 4\left(\frac{9}{2}\right)x$

$a = \frac{9}{2}$

$\boxed{x = -\frac{9}{2}}$

12

$$y^2 = 24x$$

$$y^2 = 4(6)x$$

$$a = 6$$

$$\boxed{F(6,0)}$$

13

$$x^2 - 10y^2 + 8x - 4y + 8 = 0$$

from; $Ax^2 + By^2 + Cx + Dy + E = 0$

$\boxed{\text{Hyperbola}}$

when A & B have opposite sign

14

$$y^2 + y - 4x + 8 = 0$$

$$y^2 + y + \frac{1}{4} = 4x - 8 + \frac{1}{4}$$

$$\left(y + \frac{1}{2}\right)^2 = 4x - \frac{31}{4}$$

$$\left(y + \frac{1}{2}\right)^2 = 4\left(x - \frac{31}{16}\right)$$

parabola opens to the right

$$y^2 + y - 4x + 8 = 0$$

from; $Ax^2 + By^2 + Cx + Dy + E = 0$

$\boxed{\text{parabola}}$

when A or B is zero

if $B^2 - 4AC = 0$,

then the conic equation is a $\boxed{\text{Parabola}}$

17

major axis = 10

minor axis = 8

$$e = \frac{c}{a} \quad \text{(eccentricity)}$$

$$2a = 10 \qquad 2b = 8$$
$$a = 5 \qquad b = 4$$

and; $a^2 = b^2 + c^2$

$$5^2 = 4^2 + c^2$$

$$c = 3$$

Hence;

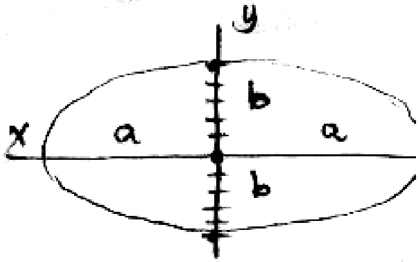
$$e = \frac{3}{5}$$

$$\boxed{e = 0.6}$$

18

$$e = \sqrt{35/50} = 0.8367$$

Minor axis $= (0, \pm 5)$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a^2 = b^2 + c^2$$

$$c = ae$$

$$a^2 = b^2 + a^2e^2$$

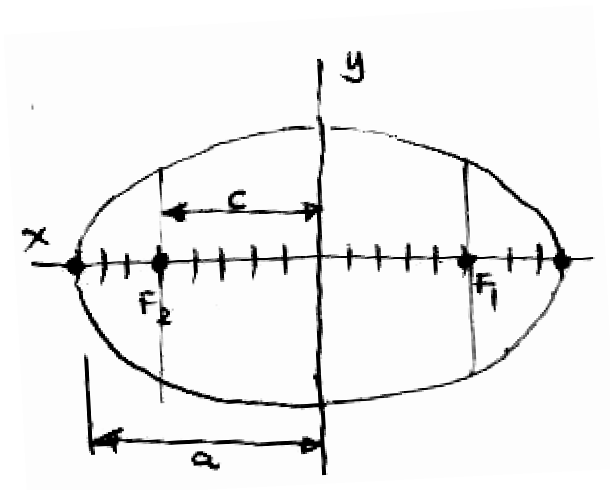
$$a^2 = 5^2 + a^2\left(\frac{35}{50}\right)$$

$$\frac{3}{10}a^2 = 25$$

$$a^2 = \frac{250}{3}$$

Hence;

$$\boxed{\frac{3x^2}{250} + \frac{y^2}{25} = 1}$$



$F(\pm 5, 0)$

MAJOR AXIS, $2a = 16$

$a = 8$

$$a^2 = b^2 + c^2$$

$$64 = b^2 + 25$$

$$b^2 = 39$$

then;

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\left[\frac{x^2}{64} + \frac{y^2}{39} = 1\right](64)(39)$$

$$\boxed{39x^2 + 64y^2 = 2496}$$

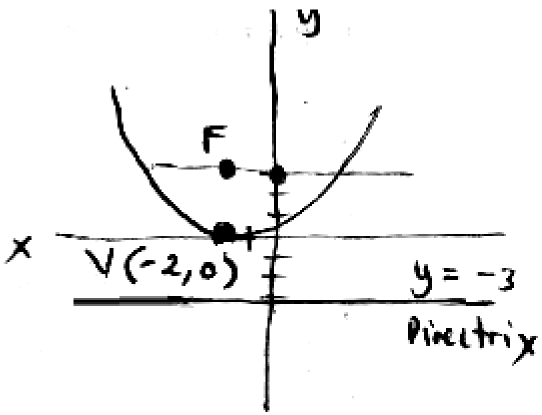
20

$$10x^2 + 18y^2 + 54x - 64y = -1$$

from: $Ax^2 + By^2 + Cx + Dy + E = 0$

$\boxed{\text{Ellipse}}$

when: $A \neq B$



Equation of parabola

F(-2,3) (Focus)

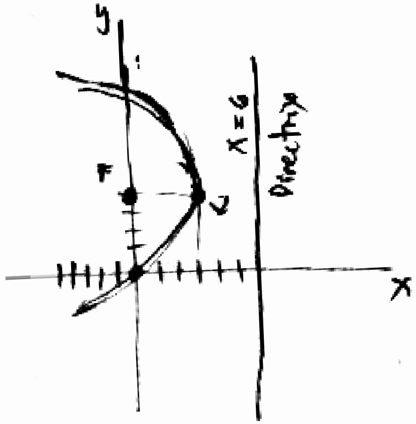
y = -3 (Directrix)

$$(x-h)^2 = 4a(y-k)$$

$$(x+2)^2 = 4(3)(y-0)$$

$$\boxed{(x+2)^2 = 12y}$$

22



V(3, 4)

directrix $x = 6$

$$(y-k)^2 = -4a(x-h)$$

$$(y-4)^2 = -4(3)(x-3)$$

$$\boxed{(y-4)^2 = -12(x-3)}$$

23

$$y^2 = 8(x-7)$$

the eccentricity of a parabola is equal to one

$$\boxed{e = 1.0}$$

Exponential LEAP ONLINE REVIEW

Angle between $V_1(4i - j + 3k)$ and $V_2(i - 3j - 5k)$

$$\cos\theta = \frac{x_2 x_1 + y_2 y_1 + z_2 z_1}{\sqrt{x_1^2 + y_1^2 + z_1^2}\ \sqrt{x_2^2 + y_2^2 + z_2^2}}$$

$$= \frac{(1)(4) + (-3)(-1) + (-5)(3)}{\sqrt{4^2 + (-1)^2 + 3^2}\ \sqrt{1^2 + (-3)^2 + (-5)^2}}$$

$$\cos\theta = -0.2652$$

$$\boxed{\theta = 105.3787^\circ}$$

$$\cos\theta = \frac{x}{\sqrt{x_1^2 + y_1^2 + z_1^2}}$$

$$\cos\theta = \frac{1}{\sqrt{1^2 + 3^2 + 1^2}}$$

$$\cos\theta = 0.3015$$

$$\boxed{\theta = 72.4516^\circ}$$

$$A_s = 4\pi r^2 = 4\pi(5m)^2$$

$$\boxed{A_s = 314.1593\ m^2}$$

27

$3x + 3y - 2z - 7 = 0$

$P(3, 3, -5)$

$$d = \frac{Ax + By + Cz + D}{\sqrt{A^2 + B^2 + C^2}}$$

$$= \frac{3(3) + 3(3) + (-2)(-5) + (-7)}{\sqrt{3^2 + 3^2 + (-2)^2}}$$

$$\boxed{d = 4.4772}$$

$$r^2 = x^2 + y^2 + z^2$$

$$r^2 = 2^2 + 6^2 + 3^2$$

$$r^2 = 49$$

$$\boxed{r = 7}$$

$P_1\ (2, -4, 10)$

$P_2\ (-5, 6, -8)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{(-5 - 2)^2 + (6 + 4)^2 + (-8 - 10)^2}$$

$$\boxed{d = 21.7486}$$

$V_1 (2,3,2)$

$V_2 (3,2,4)$

$$\cos\theta = \frac{x_2 x_1 + y_2 y_1 + z_2 z_1}{\sqrt{x_1^2 + y_1^2 + z_1^2}\,\sqrt{x_2^2 + y_2^2 + z_2^2}}$$

$$= \frac{3(2) + 2(3) + 4(2)}{\sqrt{2^2 + 3^2 + 2^2}\,\sqrt{3^2 + 2^2 + 4^2}}$$

$\cos\theta = 0.9008$

$\boxed{\theta = 25.7426^\circ}$

$P_1(6, 30^\circ)$

$P_2(3, 80^\circ)$

for P_1; $x = r\cos\theta$ $\quad$ $y = r\sin\theta$

$= 6\cos 30^\circ$ $\quad$ $= 6\sin 30^\circ$

$x_1 = 3\sqrt{3}$ $\quad$ $y_1 = 3$

for P_2; $x = r\cos\theta$ $\quad$ $y = r\sin\theta$

$= 3\cos 80^\circ$ $\quad$ $= 3\sin 80$

$x_2 = 0.5209$ $\quad$ $y_2 = 2.9544$

Hence:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0.5209 - 5.1962)^2 + (2.9544 - 3)^2}$$

$$\boxed{d = 4.6755}$$

32

$$L^2 = x^2 + y^2 + z^2$$

$$L^2 = 2^2 + 4^2 + (-5)^2$$

$$L^2 = 45$$

$$\boxed{L = 6.7082}$$

33

$$x^2 + 5y + 3xy + 4x - 3y - 12 = 0$$

A C B D E F

$$B^2 - 4AC$$

$$3^2 - 4(1)(5) = -11$$

hence; $\boxed{\text{ellipse}}$

$P_1(8, 40^\circ)$

$x = r\cos\theta$

$x = 8\cos 40^\circ$

$x = 6.1284$

$y = r\sin\theta$

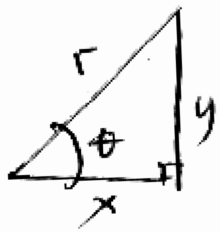
$y = 8\sin 40^\circ$

$y = 5.1423$

Hence;

$\boxed{P_1(6.1284, 5.1423)}$

35



$$x^2 + y^2 - 4x + 8y = 0$$

$$\left[r^2 - 4(r\cos\theta) + 8(r\sin\theta) = 0\right]\frac{1}{r}$$

$$r - 4\cos\theta + 8\sin\theta = 0$$

$$r - 4(\cos\theta - 2\sin\theta) = 0$$

$$\boxed{r = 4(\cos\theta - 2\sin\theta)}$$

$V = 3i - 6j + 8k$

between the x-axis

$$\cos\theta = \frac{x}{\sqrt{x_1^2 + y_1^2 + z_1^2}} = \frac{3}{\sqrt{3^2 + (-6)^2 + (8)^2}}$$

$\theta = 73.3^\circ$

between the y-axis

$$\cos\theta = \frac{y}{\sqrt{x_1^2 + y_1^2 + z_1^2}} = \frac{-6}{\sqrt{3^2 + (-6)^2 + (8)^2}}$$

$\theta = 125.0783^\circ$

betwee the z-axis

$$\cos\theta = \frac{z}{\sqrt{x_1^2 + y_1^2 + z_1^2}} = \frac{8}{\sqrt{3^2 + (-6)^2 + 8^2}}$$

$\boxed{\theta = 39.8070^\circ}$

exponential LEAP ONLINE REVIEW

$$\left[x^2 + 6y^2 = 8\right]\frac{1}{8}$$

$$\frac{x^2}{8} + \frac{6y^2}{8} = 1$$

$$\frac{x^2}{8} + \frac{y^2}{4/3} = 1$$

$$e = \frac{c}{a} \quad ; \quad a^2 = b^2 + c^2$$

$$8^2 = \left(4/3\right)^2 + c^2$$

$$c = \frac{4\sqrt{35}}{3} = 7.8881$$

Hence;

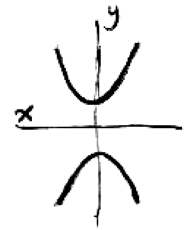
$$e = \frac{7.8881}{8}$$

$$\boxed{e = 0.9860}$$

$$\left[10x^2 - 6y^2 = -40\right] \cdot -\frac{1}{40}$$

$$-\frac{x^2}{4} + \frac{3y^2}{20} = 1$$

$$\frac{y^2}{20/3} - \frac{x^2}{4} = 1$$



$a^2 = \frac{20}{3}$; $b^2 = 4$

$a = \frac{2\sqrt{15}}{3}$; $b = 2$

$a = 2.5820$

and; $c^2 = a^2 + b^2$

$$c^2 = \frac{20}{3} + 4$$

$$c = \frac{4\sqrt{6}}{3}$$

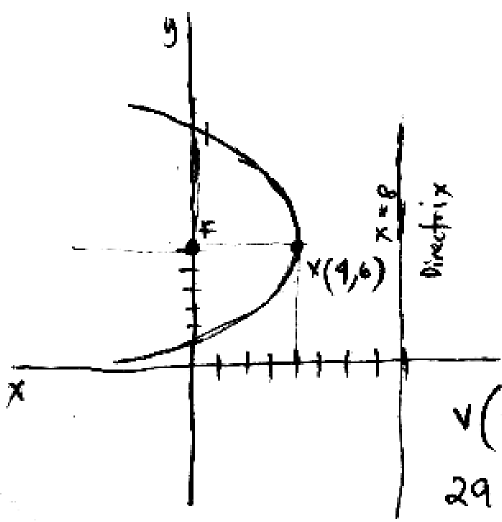
Hence; $e = \frac{c}{a} = \frac{3.2659}{2.5820}$

$$\boxed{e = 1.2649}$$

39

F (0,6)

x = 8 (directrix)



V(4,6)

$2a = 8$

$a = 4$

$$(y-k)^2 = -4a(x-h)$$

$$(y-6)^2 = -4(4)(x-4)$$

$$y^2 - 12y + 36 = -16x + 64$$

$$\boxed{y^2 - 12y + 16x - 28 = 0}$$