Answer Key

Differential Calculus

exponential LEAP ONLINE REVIEW

$$y = \sqrt{3 - x^2}$$

$$y = (3 - x^2)^{1/2}$$

$$y' = \frac{1}{2}(3 - x^2)^{-1/2}(2x)$$

$$\boxed{y' = \frac{x}{\sqrt{3 - x^2}}}$$

or use: $d\sqrt{u} = \dfrac{du}{2\sqrt{u}}$

$$y = 3(x^2-1)^{-1}$$

$$y' = -3(x^2-1)^{-2}(2x)$$

$$\boxed{y' = \frac{-6x}{(x^2-1)^2}}$$

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$$y = \frac{4-x}{x^2-2}$$

from: $d\left(\frac{u}{v}\right) = \frac{v(du) - u(dv)}{v^2}$

$$y' = \frac{[(x^2-2)(-1)] - [(4-x)(2x)]}{(x^2-2)^2}$$

$$y' = \frac{-x^2+2-8x+2x^2}{(x^2-2)^2}$$

$$\boxed{y' = \frac{x^2-8x+2}{(x^2-2)^2}}$$

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$$y = 4x^2 + 8x + 10$$

$$\boxed{y' = 8x + 8}$$

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$$\lim_{x \to 4} \frac{x^3-64}{x-4} = \frac{x^3-4^3}{x-4} = \frac{\cancel{(x-4)}(x^2+4x+16)}{\cancel{(x-4)}}$$

$$\lim_{x \to 3} (x^2+4x+16) = 4^2 + 4(4) + 16$$

$$\boxed{= 48}$$

or substitute 3.999999 to x

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$$y = 4x^3 - 4x^2 + 4x - 2 \quad @ \; P(3,6)$$

Radius of curvature: $RC = \dfrac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$

$$y' = \frac{dy}{dx} = 12x^2 - 8x + 4$$

$$y'' = \frac{d^2y}{dx^2} = 24x - 8$$

subst;

$$RC = \frac{\left[1 + (12x^2 - 8x + 4)^2\right]^{3/2}}{24x - 8} \quad @ \; x = 3$$

$$RC = \frac{\left[1 + (108 - 24 + 4)^2\right]^{3/2}}{72 - 8}$$

$$\boxed{RC = 10650.0626}$$

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$$\lim_{x \to \infty} \frac{4x-1}{10x+7} = \frac{4}{10}$$

$$\boxed{= \frac{2}{5}}$$

$$y = \frac{1-3x^2}{x-3} \quad @ \; x = 1$$

using calcu: $\left.\dfrac{d}{dx}\left(\dfrac{1-3x^2}{x-3}\right)\right|_{x=1}$

$$\boxed{= 3.5}$$

partial derivatives:

$$x^2 + 6x + 10z^2$$

$$\boxed{\frac{\partial M}{\partial x} = 2x + 6}$$

partial derivatives:

$$5y^2 - 6y + 8$$

$$\boxed{\frac{\partial M}{\partial y} = 10y - 6}$$

partial derivatives:

$$3x^2 - 4xy$$

$$\frac{\partial M}{\partial x} = 6x - 4\left[x(0) + y(1)\right]$$

$$\boxed{\frac{\partial M}{\partial x} = 6x - 4y}$$

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$$y = x + 3x^{1/4}$$

$$m = \frac{dy}{dx} = 1 + \frac{3}{4}x^{-3/4} = 1 + \frac{3}{4}(9)^{-3/4} = 1.1443 \approx \frac{8}{7}$$

from: $y - k = m(x - h)$

$$y - 13 = \frac{8}{7}(x - 9)$$

$$7y - 91 = 8x - 72$$

$$\boxed{8x - 7y + 19 = 0}$$

$$(x+2)^3 - x^3$$

conventional:

$$3(x+2)^2(1) - 3x^2$$

$$3(x^2+4x+4) - 3x^2$$

$$\cancel{3x^2} + 12x + 12 - \cancel{3x^2}$$

$$12x + 12$$

$$\boxed{12(x+1)}$$

using calcu:

$$\frac{d}{dx}\left[(x+2)^3 - x^3\right]\Big|_{x=1} = 24$$

try: $12(x+1)$

$12(2) = 24$

$y = \sec(x^2+3)$

using calcu:

Setting: radians

$$\frac{d}{dx}\left[\frac{1}{\cos(x^2+3)}\right]\Bigg|_{x=1} = -3.5427$$

try: $\boxed{2x\sec(x^2+3)\tan(x^2+3)}$

$$2(1)\sec[(1)^2+3]\tan[(1)^2+3] = -3.5427$$

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exponential LEAP ONLINE REVIEW

$$f(x) = \left[x^4 - (x-1)^3\right]^3$$

conventional:

$$3\left[x^4 - (x-1)^3\right]^2 4x^3 - 3(x-1)^2(1)$$

$$3\left[x^4 - (x-1)^3\right]^2 4x^3 - 3(x^2 - 2x + 1)$$

$$\boxed{3\left[x^4 - (x-1)^3\right]^2 (4x^3 - 3x^2 + 6x - 3)}$$

using calcu:

$$\left.\frac{d}{dx}\left[x^4 - (x-1)^3\right]^3\right|_{x=1} = 12$$

try: $3\left[x^4 - (x-1)^3\right]^2 (4x^3 - 3x^2 + 6x - 3)$

$$3(1)^2(4 - 3 + 6 - 3) = 12$$

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$$y = -3x^2$$

$$m = \frac{dy}{dx} = -6x \text{ @ } P(1,4)$$

$$m = -6(1)$$

$$\boxed{m = -6}$$

$$x(t) = t^3 - 4t^2 - 40t + 60$$

$$v = \frac{dx}{dt} = 3t^2 - 8t - 40$$

$$v(t) = 0 = 3t^2 - 8t - 40$$

$$t_1 = 5.2206 \text{ sec}$$

$$t_2 = -2.5540 \text{ sec (disregard)}$$

and;

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 6t - 8$$

$$a = 6(5.2206 \text{ sec}) - 8$$

$$\boxed{a = 23.3236 \text{ ft/sec}^2}$$

$$x(t) = 3t^3 - 10t^2 - 45t + 50$$

$$v = \frac{dx}{dt} = 9t^2 - 20t - 45$$

and;

$$a = \frac{dv}{dt} = 18t - 20$$

$$a = 0 = 18t - 20$$

$$a = 1.1111 \text{ sec}$$

Hence;

$$v = 9(1.1111\text{sec})^2 - 20(1.1111\text{sec}) - 45$$

$$\boxed{v = -56.1111 \text{ m/s}}$$

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$$y = 14x^2 + 4$$

$$m = \frac{dy}{dx} = 28x \quad @ \quad x = 2$$

$$m = 28(2)$$

$$\boxed{m = 56}$$

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$$x = 4t^3 + 3t^2 - t + 4$$

$$v = \frac{dx}{dt} = 12t^2 + 6t - 1$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 24t + 6$$

$$@\ t = 3$$

$$a = 24(3) + 6$$

$$\boxed{a = 78}$$

$$y = 4x^3 - 5x^2 - 25x + 26$$

$$y' = \frac{dy}{dx} = 12x^2 - 10x - 25$$

$$y'' = \frac{d^2y}{dx^2} = 24x - 10 = 0$$

$$24x = 10$$

$$x = \frac{5}{12}$$

Hence;

$$y = 4\left(\frac{5}{12}\right)^3 - 5\left(\frac{5}{12}\right)^2 - 25\left(\frac{5}{12}\right) + 26$$

$$\boxed{y = 15}$$

$$y = 3x^3 - x^2 - 20x + 20$$

$$y' = \frac{dy}{dx} = 9x^2 - 2x - 20 = 0$$

$$x_1 = 1.6059$$

$$x_2 = -1.3837$$

Hence;

$$y = 3(1.6059)^3 - (1.6059)^2 - 20(1.6059) + 20$$

$$\boxed{y = -2.2725}$$

$x + y = 60$

$xy = \text{maximum}$

$(30)(30) = 900$

choose: $\boxed{30 \text{ and } 30}$

$$\frac{dr}{dt} = 0.20 \ \text{in/sec}$$

$$A = \pi r^2 \ ; \ r = 5 \ \text{in}$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$= 2\pi (5\text{-in})(0.20 \ \text{in/sec})$$

$$\boxed{\frac{dA}{dt} = 2\pi}$$

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maximum area of a rectangle:

$$A = \frac{P^2}{16} = \frac{(150 in)^2}{16}$$

$$\boxed{A = 1406.25 \; in^2}$$

exponential LEAP ONLINE REVIEW

maximum area of a triangle:

$$A = \frac{1}{18} P^2 \sin 60^\circ$$

$$= \frac{1}{18} (40\ in)^2 \sin 60^\circ$$

$$\boxed{A = 76.98\ in^2}$$

exponential LEAP ONLINE REVIEW

$A = 4\pi r^2$ (surface area)

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$ (eq 1)

from: $V = \frac{4}{3}\pi r^3$

$130\ ft^3 = \frac{4}{3}\pi r^3$

$r = 3.1426\ ft$

differentiating V;

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$2.4\ \frac{ft^3}{sec} = 4\pi(3.1426\ ft)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = 0.01934\ ft/sec$

subst; to eq. 1 :

$\frac{dA}{dt} = 8\pi(3.1426\ ft)(0.01934\ ft/sec)$

$\boxed{\frac{dA}{dt} = 1.5274\ \frac{ft^2}{sec}}$

$x(t) = 3\sin t$

$y(t) = 2\cos t$

resultant @ $t = \pi$

$$\frac{dx}{dt} = 3\cos t$$

$$\frac{dy}{dt} = -2\sin t$$

and: $V = \sqrt{V_x^2 + V_y^2}$

$$= \sqrt{[3\cos(\pi)]^2 + [-2\sin(\pi)]^2}$$

$$\boxed{V = 3.0}$$

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exponential LEAP ONLINE REVIEW

$$f(x) = 2x^3 - 4x - 6$$

$$y' = \frac{dy}{dx} = 6x^2 - 4 = 0$$

$$x^2 = \frac{4}{6} = \frac{2}{3}$$

$$x = \pm\frac{\sqrt{6}}{3} = \pm 0.8165$$

substitute x;

for $+\frac{\sqrt{6}}{3}$; $f(x) = -8.1773$

for $-\frac{\sqrt{6}}{3}$; $= f(x) = -3.8227$

Hence; minimum value of the function is: $\boxed{\frac{\sqrt{6}}{3}}$

$$y = 3x^3 - 20x + 16$$

$$y' = \frac{dy}{dx} = 9x^2 - 20 = 0$$

$$x^2 = \frac{20}{9}$$

$$x = \pm \frac{2\sqrt{5}}{3} = \pm 1.4908$$

substitute x;

for $+\frac{2\sqrt{5}}{3}$; $f(x) = -3.8762$

for $-\frac{2\sqrt{5}}{3}$; $f(x) = 35.8762$

Hence; minimum value of y is: $\boxed{\frac{2\sqrt{5}}{3} = 1.4908}$

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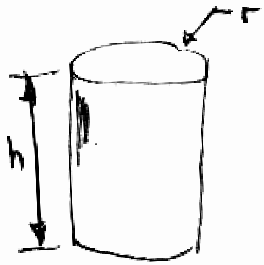

$$r = \sqrt[3]{\frac{V}{2\pi}}$$

$$V = 8\,L\left(\frac{1\,m^3}{1000\,L}\right)\left(\frac{100\,cm}{1\,m}\right)^3 = 8000\,cm^3$$

$$r = \sqrt[3]{\frac{8000\,cm^3}{2\pi}}$$

$$\boxed{r = 10.8385\,cm}$$

and;

$$h = 2r = 2(10.8385\,cm)$$

$$\boxed{h = 21.6770\,cm}$$

$xy = 18$

$x + y^2 = \text{Least}$

trial & error

$(9)(2) = 18$

$(9) + (2)^2 = 13$

choose: $\boxed{9 \text{ and } 2}$

$y = e^x \cos x^3$

using calcu:

setting: radians

$$\frac{d}{dx}\left[e^x \cos x^3\right]\Big|_{x=1} = -5.3934$$

try: $\boxed{e^x(\cos x^3 - 3x \sin x^3)}$

$$e^1\left[\cos(1)^3 - 3(1)\sin(1)^3\right] = -5.3934$$