Answer Key

Integral Calculus and Differential Equations

1

$$\int_{-2}^{2} (5x^3 - 4x^2)\,dx$$

$$= \frac{5x^4}{4} - \frac{4x^3}{3} \Big|_{-2}^{2}$$

$$= \left(20 - \frac{32}{3}\right) - \left(20 + \frac{32}{3}\right)$$

$$\boxed{= \frac{-64}{3}}$$

using calcu:

$$\int_{-2}^{2} (x^3 - 4x^2)\,dx = -\frac{64}{3}$$

2

$$\int \frac{x}{x^2+2}\,dx$$

let $u = x^2 + 2$

$du = 2x\,dx$

$\frac{du}{2} = dx$

$$= \frac{1}{2}\int \frac{du}{u}$$

$$= \frac{1}{2}\ln u + C$$

$$\boxed{= \frac{1}{2}\ln(x^2+2) + C}$$

or; using calcu, and assign 1 and 0 as upper and lower limits

$$\int_0^1 \frac{x}{x^2+2}\,dx = 0.2027$$

substitute to choices: $\frac{1}{2}\ln(x^2+2) + C$

$$= \frac{1}{2}\left[\ln(1+2) - \ln(0+2)\right] = 0.2027$$

3

$$\int \sqrt{2-x}$$

$$= \int (2-x)^{1/2}\, dx$$

let $u = (2-x)$

$du = dx$

$$= \int u^{1/2}\, du$$

$$= \frac{2u^{3/2}}{3} + C$$

$$\boxed{= \frac{2}{3}(2-x)^{3/2} + C}$$

4

$y^2 = 10x$

$x = y$

point of intersection

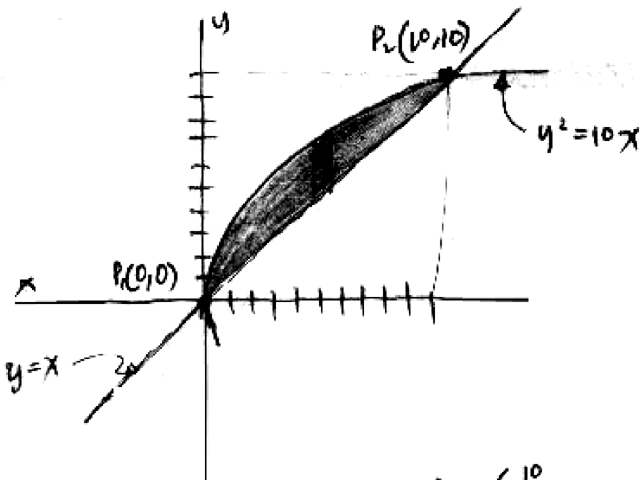
$y^2 = 10y$

$y^2 - 10y = 0$

$y(y-10) = 0$

$y = 0$

$y = 10$



$$A = \int_0^{10} (y_1 - y_2)\,dx$$

$$= \int_0^{10} \left(\sqrt{10x} - x\right) dx$$

$$\boxed{A = \frac{50}{3}}$$

5

$y = 0$

$y = e^x$

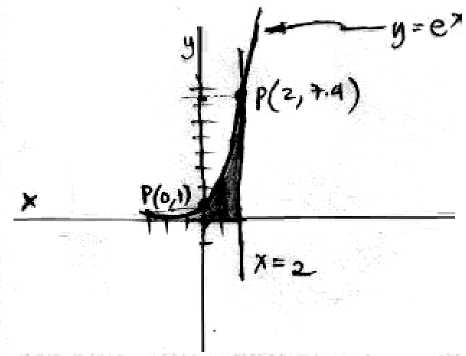
$x = 0$

$x = 2$

point of intersection:

$x = 0 \; ; \; y = 1$

$x = 2 \; ; \; y = 7.3891$



$$A = \int_0^2 y\,dx$$

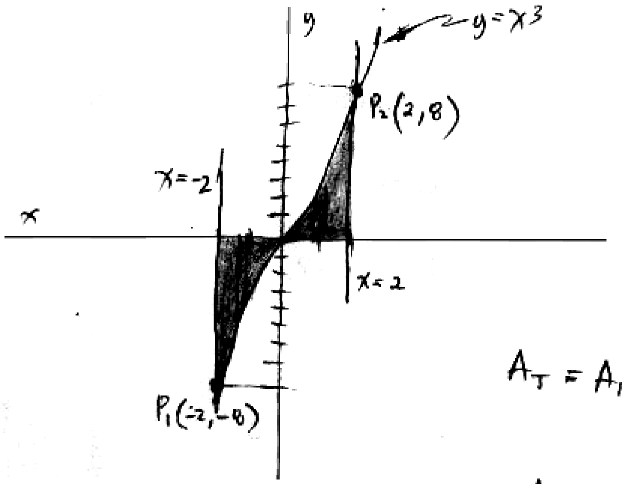
$$= \int_0^2 e^x\,dx$$

$$\boxed{A = 6.3891 \text{ sq. units}}$$

6

$y = x^3$
x-axis
$x = -2$
$x = 2$

point of intercection
$x = -2 \;;\; y = -8$
$x = 2 \;;\; y = 8$



$A_T = A_1 + A_2$
but ; $A_1 = A_2$

$A_T = 2A$

$= 2\int_0^2 y\,dx$

$= 2\int_0^2 x^3\,dx$

$\boxed{A_T = 8 \text{ sq. units}}$

7

$x^2 = y$
$y = 3x^2 - 6x$

point of intersection

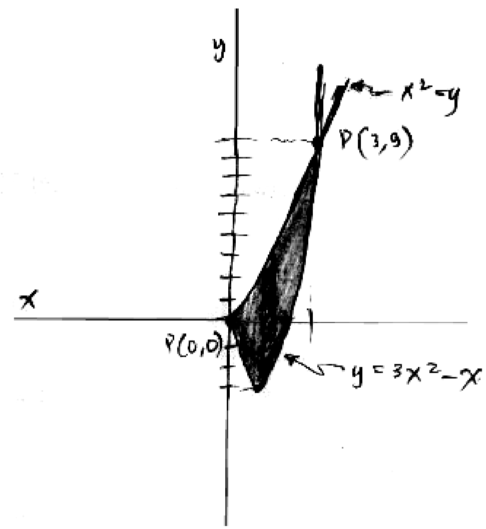
$x^2 = 3x^2 - 6x$

$6x = 2x^2$

$2x^2 - 6x = 0$

$2x(x-3) = 0$

$x = 0 \;;\; y = 0$
$x = 3 \;;\; y = 9$



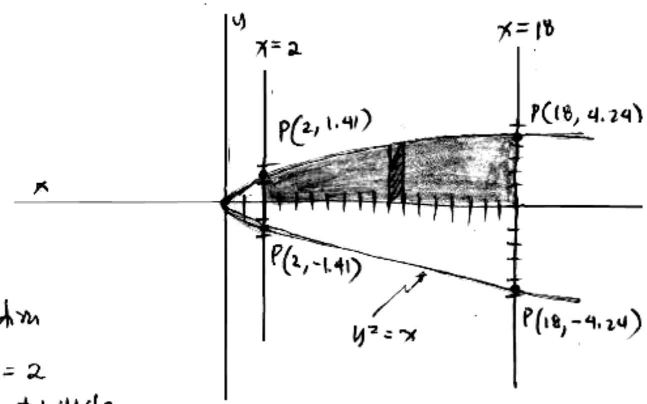
$$A = \int_0^3 (y_{P_1} - y_{P_2})\,dx$$

$$= \int_0^3 \left[x^2 - (3x^2 - 6x)\right] dx$$

$\boxed{A = 9 \text{ sq. units}}$

8

$y^2 = x$
$x = 2$
$x = 18$
$x =$ axis



Point of intersection

$x = 2 \qquad y^2 = 2$
$\qquad\qquad y = \pm 1.4142$

$x = 18 \qquad y^2 = 18$
$\qquad\qquad y = \pm 4.2426$

$A = A_1 - A_2$

$A_1 = \frac{2}{3} LD$

$= \frac{2}{3}(4.24)(18)$

$A_1 = 50.88$ sq. units

$A_2 = \frac{2}{3} LD$

$= \frac{2}{3}(1.41)(2)$

$A_2 = 1.88$ sq. units

substi

$A = (50.88 - 1.88)$ sq. units

$\boxed{A = 49 \text{ sq. units}}$

$y = x^2$

$x = 4$

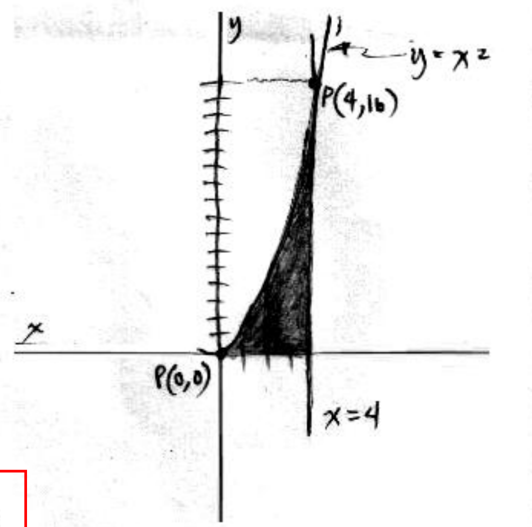
point of intersection:

$x = 4$; $y = 16$

$A = \int_0^4 y_p \, dx$

$A = \int_0^4 x^2 \, dx$

$\boxed{A = 21.3333 \text{ sq. units}}$



10

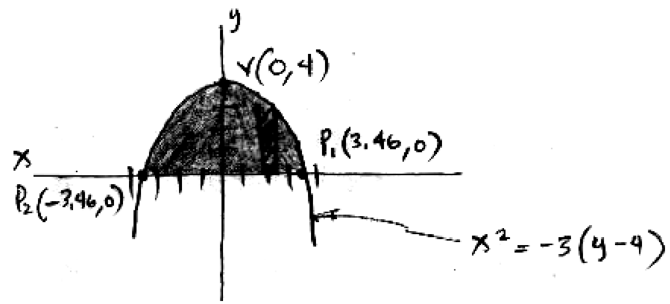
$x^2 = -3y + 12$

$x^2 = -3(y-4)$

$V(0,4)$

$x^2 = 12$

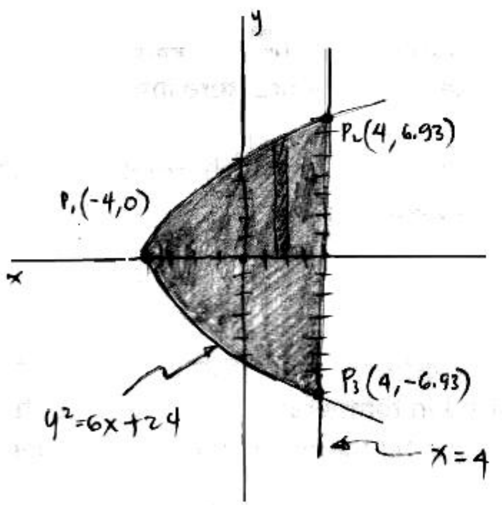
$x = \pm 3.46$



$A = \frac{2}{3} L D$

$= \frac{2}{3}(3.46)(2)(4)$

$\boxed{A = 18.4533 \text{ sq. units}}$



$y^2 = 6x + 24$

$y^2 = 6(x + 4)$

$V = (0, -4)$

$x = 0;\ y = \pm 4.8990$

$x = 4$ points of intersection

$y^2 = 6(4) + 24$

$y = \pm 6.9282$

$A = \frac{2}{3} LD$

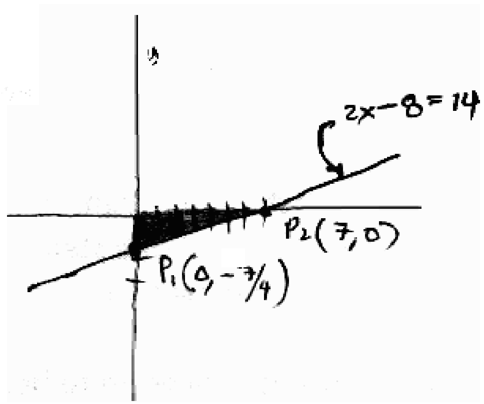
$= \frac{2}{3}(6.93)(2)(8)$

$\boxed{A = 73.92 \text{ sq. units}}$

12

$\left[2x - 8y = 14\right]\frac{1}{14}$

$\frac{x}{7} + \frac{y}{\left(-\frac{7}{4}\right)} = 1$



Area of triangle:

$A = \frac{1}{2} b h$

$= \frac{1}{2}(1.75)(7)$

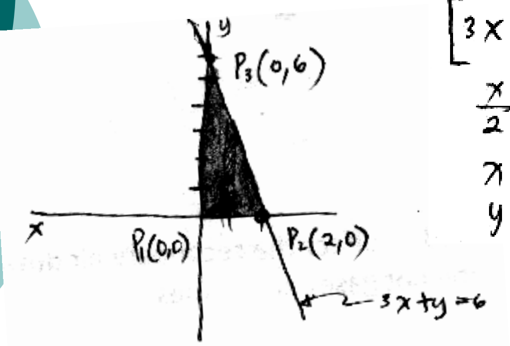
$\boxed{A = 6.125 \text{ sq. units}}$

or

$A = \int_7^0 y\,dx = \int_0^7 \left(\frac{2x-14}{8}\right) dx$

$A = 6.125$ sq. units

13



$$\left[3x+y=6\right]\frac{1}{6}$$

$$\frac{x}{2}+\frac{y}{6}=1$$

$$x=0$$

$$y=0$$

(vertical strip) $x_c = x$; $y_c = \frac{y}{2}$

$$\int dA = \int y\,dx$$

$$A=\int_0^2 (6-3x)\,dx$$

$$A=6$$

from;

$$A\bar{x}=\int_0^2 x_c\,dA$$

$$A\bar{x}=\int_0^2 x_c\,y\,dx$$

$$A\bar{x}=\int_0^2 x(6-3x)\,dx=4$$

subst;

$$6\bar{x}=4$$

$$\bar{x}=\frac{4}{6}=\frac{2}{3}$$

from;

$$A\bar{y}=\int_0^2 y_c\,dA$$

$$A\bar{y}=\int_0^2 \frac{y}{2}(6-3x)\,dx$$

$$A\bar{y}=\int_0^2 \left(\frac{6-3x}{2}\right)(6-3x)\,dx=12$$

$$A\bar{y}=12$$

$$6\bar{y}=12$$

$$\bar{y}=2$$

Hence;

$$\boxed{\text{centroid } \frac{2}{3}, 2}$$

exponential LEAP
ONLINE REVIEW

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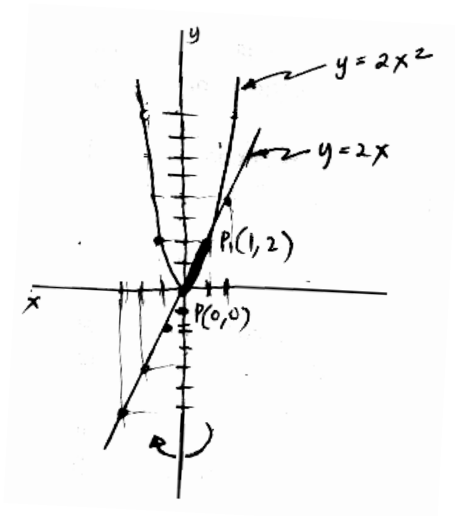


$y = 2x^2$

$y = 2x$

revolved at y-axis

solving for point of intersection

$$2x = 2x^2$$

$$2x^2 - 2x = 0$$

$$2x(x-1) = 0$$

$x = 0 \qquad x = 1$

$y = 0 \qquad y = 2$

$$V = \pi \int_0^2 \left(x_p^2 - x_l^2\right) dy$$

$$= \pi \int_0^2 \left[\left(\frac{y^{1/2}}{2^{1/2}} \right)^2 - \left(\frac{y}{2} \right)^2 \right] dy$$

$$= \pi \int_0^2 \left[\frac{y}{2} - \frac{y^2}{4} \right] dy$$

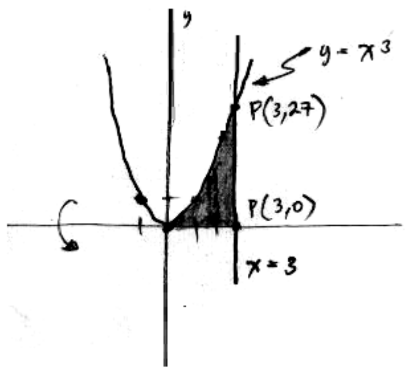
$$\boxed{V = \frac{1}{3}\pi \text{ cubic units}}$$

15

$y = x^3$
$y = 0$
$x = 3$

$y = x^3$

x	-4	-3	-2	-1	0	1	2	3	4
y	-64	-27	-8	-1	0	1	8	27	64



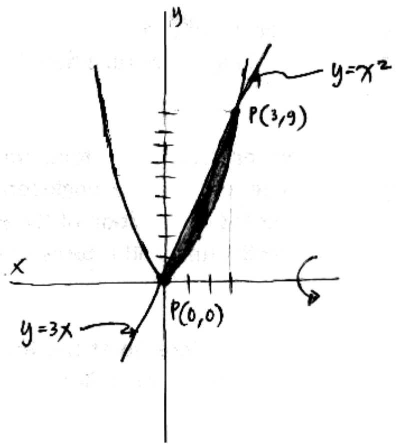
$$V = \pi \int_0^3 (y_P^2 - y_L^2)\,dx$$

$$= \pi \int_0^3 \left[(x^3)^2 - 0 \right] dx$$

$$V = \frac{2187}{7}\pi$$

$$\boxed{V = 981.5233 \text{ cu. units}}$$

16



$y = x^2$

$y = 3x$

point of intersection:

$3x = x^2$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \quad x = 3$

$y = 0 \quad y = 9$

$$V = \pi \int_0^3 (y_p^2 - y_L^2)\,dx$$

$$= \pi \int_0^3 \left[(x^2)^2 - (3x)^2\right] dx$$

$$V = \frac{162}{5}\pi$$

$$\boxed{V = 101.7876 \text{ cu. units}}$$

17

$$3y\,dx + (xy + 6x)\,dy = 0$$

$$3y\,dx + (y+6)x\,dy = 0$$

$$\left[3y\,dx = -(y+6)x\,dy\right]\frac{1}{x}\,\frac{1}{y}$$

$$3\frac{dx}{x} = -(y+6)\frac{dy}{y}$$

$$3\frac{dx}{x} = -dy - 6\frac{dy}{y}$$

$$3\int\frac{dx}{x} = -\int dy - 6\int\frac{dy}{y}$$

$$3\ln x = -y - 6\ln y + c$$

$$\ln x^3 + y + \ln y^6 = c$$

$$\ln(x^3)(e^y)(y^6) = c$$

$$e^{\ln x^3 e^y y^6} = e^c$$

$$\boxed{x^3 y^6 e^y = c}$$

$$x\,dy + y\,dx = 0$$

$$\left[x\,dy = -y\,dx\right]\frac{1}{x}\,\frac{1}{y}$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\ln y = -\ln x + c$$

$$\ln y + \ln x = c$$

$$\ln (y)(x) = c$$

$$e^{xy} = e^{c}$$

$$\boxed{xy = C}$$

19

$$3xy + 9x + (x^2-5)y' = 0$$

$$\left[3xy + 9x + (x^2-5)\frac{dy}{dx} = 0\right]dx$$

$$3xy\,dx + 9x\,dx + (x^2-5)\,dy = 0$$

$$\left[(y+3)3x\,dx + (x^2-5)\,dy = 0\right]\left(\frac{1}{y+3}\right)\left(\frac{1}{x^2-5}\right)$$

$$\int \frac{3x\,dx}{x^2-5} = -\int \frac{dy}{y+3}$$

let $u = x^2-5$ $\quad$ let $z = y+3$

$du = 2x\,dx$ $\quad$ $dz = dy$

$$\frac{3}{2}\int \frac{du}{u} = -\int \frac{dz}{z}$$

$$\frac{3}{2}\ln u = -\ln z + c$$

$$\frac{3}{2}\ln(x^2-5) = -\ln(y+3) + c$$

$$\frac{3}{2}\ln(x^2-5) + \ln(y+3) = c$$

$$\ln\left[(x^2-5)^{3/2}(y+3)\right] = c$$

$$e^{\ln\left[(x^2-5)^{3/2}(y+3)\right]} = e^c$$

$$\boxed{(x^2-5)^{3/2}(y+3) = C}$$

$y' = 3x$

$y = 8$ if $x = 2$

$$\frac{dy}{dx} = 3x$$

$$\int dy = 3\int x\,dx$$

$$y = \frac{3x^2}{2} + c$$

when; $y = 8$ & $x = 3$

$$8 = \frac{3(2)^2}{2} + c$$

$$C = 2$$

Hence;

$$y = \frac{3x^2}{2} + 2$$

$$2y = 3x^2 + 4$$

$$\boxed{3x^2 - 2y + 4 = 0}$$