ENGINEERING MECHANICS

Statics of Rigid Bodies

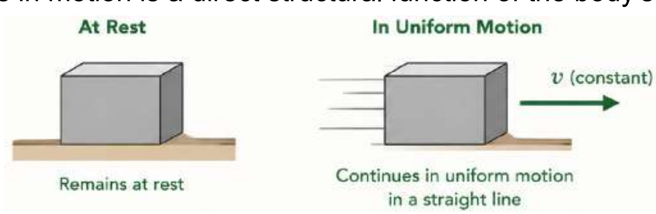
Statics is a primary branch of engineering mechanics that investigates the effects of forces acting upon particles or rigid bodies that are entirely at rest or moving at a constant velocity in a state of continuous equilibrium. Unlike dynamic systems where unbalanced forces cause varying accelerations, static systems deal with balanced internal and external force interactions. In analyzing these engineering configurations, mechanical quantities are divided into two fundamental mathematical categories based on their dimensional attributes:

- **Vector Quantities:** These are physical parameters that possess both a defined magnitude and a specific spatial direction. Within structural and mechanical analysis, critical examples include forces, velocities, accelerations, and displacements.
- **Scalar Quantities:** These are physical parameters characterized exclusively by their magnitude, requiring no spatial direction for a complete mathematical description. Standard engineering examples include mass, time, temperature, and volume.

Newton's Laws of Motion

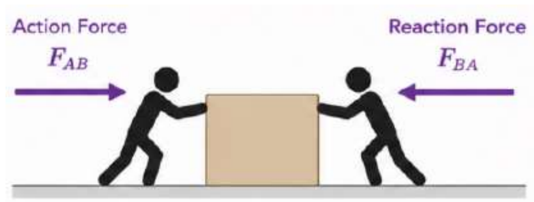
The foundational mechanics governing both static and dynamic systems are anchored by the three universal laws formulated by Sir Isaac Newton. These principles define how mass interacts with external inputs:

1. **First Law of Motion (Law of Inertia):** This law dictates that an engineering body will remain in a permanent state of rest or maintain a continuous, uniform motion in a straight line unless it is compelled to change that state by an external, unbalanced force. This tendency to resist changes in motion is a direct structural function of the body's mass.



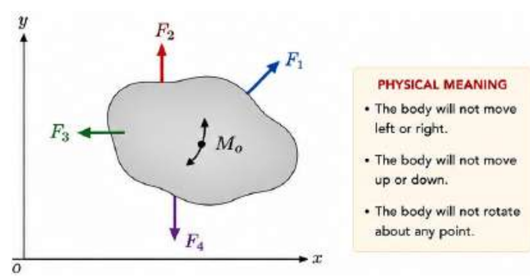
2. **Second Law of Motion (Law of Acceleration):** This principle states that when a body of mass m is subjected to an external unbalanced force F, it will experience an instantaneous acceleration. This acceleration is directly proportional to the magnitude of the applied force, inversely proportional to the mass of the body, and acts along the exact same line of action and direction as the applied force. Mathematically, it is modeled by the foundational formula: $F = ma$

3. **Third Law of Motion (Law of Action-Reaction):** This law establishes that for every applied force action, there is an simultaneous, equivalent force reaction that is exactly equal in magnitude but oriented in the directly opposite direction. Within a static structure, these action-reaction pairs manifest as structural constraints, support reactions, and internal stresses that keep the components unified.



Conditions for Static Equilibrium

For any rigid body to achieve a state of complete static equilibrium within a two-dimensional or three-dimensional space, the resultant of all external forces and moments acting upon it must equal zero. In a standard planar (2D) coordinate framework, this absolute stability requires satisfying three distinct scalar equations of equilibrium simultaneously:



$$\Sigma F_x = 0$$

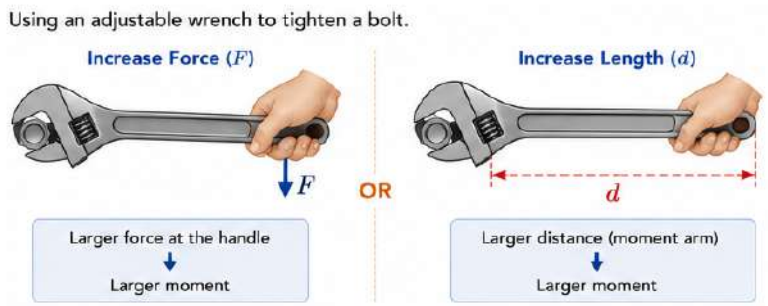
$$\Sigma F_y = 0$$

$$\Sigma M_o = 0$$

The first two conditions ($\Sigma F_x = 0$ and $\Sigma F_y = 0$) dictate that the algebraic sum of all horizontal and vertical forces must balance out perfectly, preventing any linear translational acceleration along the x or y axes. The third condition ($\Sigma M_o = 0$) ensures that the net rotational effect about any arbitrarily selected pivot point o evaluates to zero, preventing any angular rotational acceleration.

Concept of a Moment

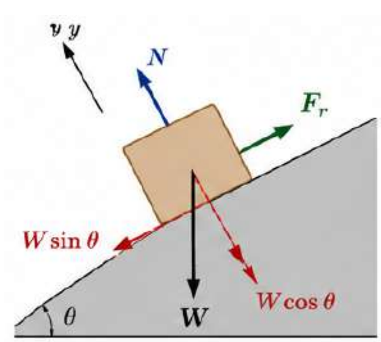
A **moment** (M_o) represents the formal measure of the capacity or structural ability of an applied force to cause a rigid body to rotate about a specific axis or reference point o. Geometrically, the magnitude of a moment is equal to the product of the magnitude of the applied force (F) and the shortest, perpendicular distance (d) measured from the chosen reference axis or pivot point to the line of action of that force. This perpendicular distance is commonly referred to in engineering mechanics as the **moment arm** or lever arm: $M_o = F \cdot d$



In mechanical operations, such as utilizing an adjustable wrench to tighten a bolt, the moment can be maximized either by increasing the linear force applied at the handle or by extending the total length d of the wrench body.

Mechanics of Friction on an Inclined Plane

Friction is a fundamental resistive force that acts along the contact interface between two contacting bodies, operating in a direction that directly opposes relative sliding motion or the immediate tendency toward motion. When analyzing a block of weight W resting on a surface inclined at an angle θ, the system experiences two primary regimes of friction:



- **Static Friction:** This represents the variable resistive force that prevents an object from slipping when a force is applied. It reaches its peak value at the exact threshold of impending motion, where the object is on the verge of sliding or "starts to move."
- **Kinetic Friction:** This is the constant resistive force that acts against the contact interface once relative motion is established and the object is already moving, serving to maintain or damp its current velocity profile.

Derivation of the Coefficient of Friction (f)

To isolate the fundamental relationship governing friction, a block of weight W is evaluated on an inclined plane at the precise angle θ where slipping is impending. By establishing a localized coordinate axis system where the x-axis runs parallel to the incline and the y-axis runs perpendicular to the surface, the weight vector W is broken down into rectangular components ($W\ sin\theta$ acting down the plane, and $Wcos\theta$ acting directly into the plane).

Applying the first equilibrium condition parallel to the incline ($\Sigma F_x = 0$, defining up the incline as positive): $F_r - W\ sin\theta = 0$

$$F_r = W\ sin\theta \qquad \text{(eq.1)}$$

Applying the second equilibrium condition perpendicular to the incline ($\Sigma F_y = 0$, defining upwards from the surface as positive): $N - W\,cos\theta = 0$

$$W = \frac{N}{cos\theta} \qquad \text{(eq.2)}$$

Where F_r represents the total frictional force, and N represents the normal force that acts perpendicular to the contact boundary. Substituting Equation 2 directly into Equation 1 eliminates the weight parameter, linking the frictional force directly to the normal reaction force:

$$F_r = N\left(\frac{sin\theta}{cos\theta}\right)$$

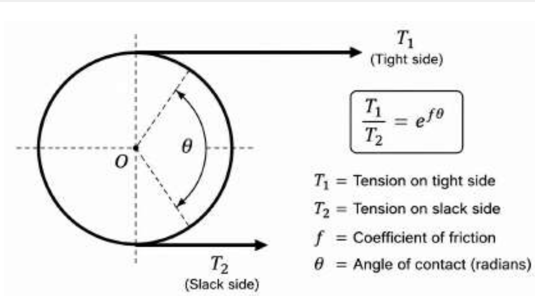
$$F_r = N\,tan\theta$$

By fundamental definition, the ratio of the maximum static frictional force to the corresponding normal force is a constant known as the **coefficient of friction** (f). Therefore, dividing both sides by N demonstrates that the tangent of this critical angle of impending slip matches the coefficient of friction exactly: $tan\theta = \frac{F_r}{N} = f$

Rearranging this relationship yields the classic engineering design equation for calculating the total available frictional resistance within a mechanical joint or interface: $F_r = fN$

Belt Friction

When a flexible band, rope, or belt wraps around a curved surface such as a pulley, cylinder, or bollard, the friction developed along the contact interface allows for a significant difference in tension between the two sides of the belt. This phenomenon is essential for analyzing belt-driven machinery, brakes, and marine mooring systems. The mechanical relationship governing this transmission of force is defined by the classic belt friction equation:



In this structural formulation, T_1 represents the tension on the tight side, while T_2 represents the tension on the slack side, meaning T_1 must always be numerically greater than T_2 when slippage is impending or occurring ($T_1 > T_2$). The parameter f is the non-dimensional coefficient of static or kinetic friction between the flexible belt material and the curved contact surface. The variable θ represents the total angle of contact, also known as the wrap angle or angle of lap.

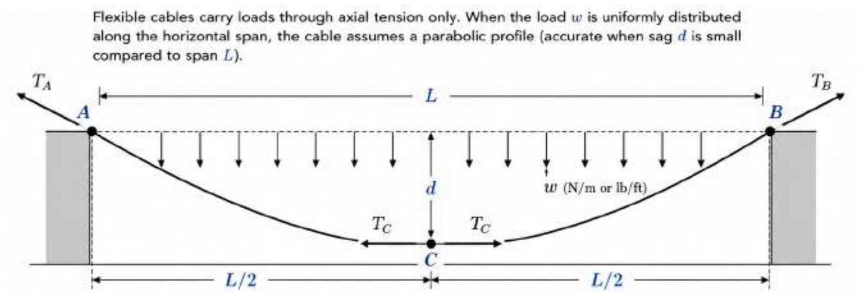
Note: The wrap angle θ must strictly be converted and evaluated in **radians** within this exponential formula. Utilizing degrees instead of radians is one of the most common mistakes made during examinations and will lead to an incorrect numerical result.

Structural Analysis of Flexible Cables

Flexible cables are utilized extensively in engineering structures, such as suspension bridges, transmission lines, and guy wires, to support loads across expansive spans. Because cables possess negligible bending stiffness, they carry external loads purely through internal axial tension. The structural profile assumed by a suspended cable depends entirely on how its loading is distributed across space:

1. Parabolic Cables

A cable takes on a parabolic profile when its load w (expressed in structural units such as N/m or lb/ft) is assumed to be uniformly distributed along a horizontal projection line. This assumption is highly accurate for suspension bridges where the heavy roadway deck dominates the system weight. Furthermore, a cable can be mathematically approximated by a parabola when its maximum sag d at mid-span is significantly small compared to its total horizontal span length L. Under these horizontal conditions, the structural tensions and total cable length are evaluated using the following expressions:



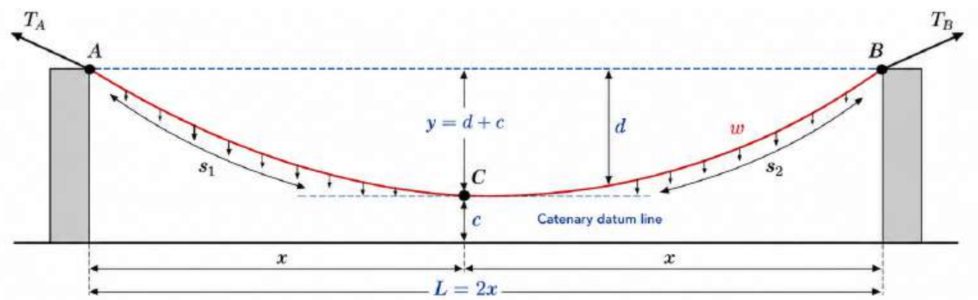
- **Tension at the Lowest Point (C):** The minimum internal tension occurs precisely at the lowest point of the curve (C), where the cable slope is perfectly horizontal: $T_C = \frac{wL^2}{8d}$

- **Tension at the Supports (A and B):** The maximum internal tension occurs at the high-elevation boundary supports (A and B), where the cable slope is steepest. This maximum value is the vector resultant of the minimum horizontal tension T_C and the total vertical shear load acting on half the span $\left(\frac{wL}{2}\right)$: $T_A = T_B = \sqrt{\left(\frac{wL}{8d}\right)^2 + T_C^2}$

- **Approximate Total Length of the Cable (S):** Because a true parabolic arc length formula involves complex logarithmic series, structural engineers utilize a highly accurate power series approximation to determine the total curved length S based on the span L and sag d: $S = L + \frac{8d^2}{3L} - \frac{32d^4}{5L^3}$

2. Catenary Cables

A cable takes on a true catenary profile when the load w is distributed uniformly along the actual curved arc length of the cable itself rather than a horizontal line. This structural configuration represents a heavy, flexible cable or chain hanging freely under the sole influence of its own dead weight. This model must be utilized when the sag d is very large relative to the horizontal span L, as the horizontal projection approximation loses its accuracy.



To mathematically analyze a catenary, an imaginary horizontal datum line is established at a distance c directly below the lowest point of the cable. This parameter c represents the parameter of the catenary, creating a total vertical coordinate height y = d + c at the support anchors. The underlying geometric and force relationships are governed by the following equations:

- **Maximum Support Tension:** The maximum internal tension occurs at supports A and B and is directly proportional to the total vertical distance y measured from the imaginary catenary datum line: $T_A = T_B = wy$
- **Geometric Arc Length Relationships:** The geometric relationship between the vertical height y, the partial curved cable arc length from the center to a support (s_1 or s_2), and the catenary constant c forms a right-triangle power relationship: $y^2 = s^2 + c^2$
- **Total Cable Length (S):** The total length of the suspended cable is simply the sum of its symmetrical left and right curved sections: $S = s_1 + s_2$
- **Horizontal Span Distance (L):** The total horizontal span distance between the two matching support towers is calculated using hyperbolic or natural logarithmic functions that track the spatial profile: $L = 2x = 2c\, ln\left(\frac{s+y}{c}\right)$

Properties of Sections and Areas

In structural engineering and machine component design, analyzing how an area is geometrically distributed around a reference axis is essential for predicting structural strength and deflection behavior under mechanical loads.

Foundational Geometric Formulations

1. Area Moment of Inertia (I)

The area moment of inertia, or second moment of area, measures an area's structural resistance to bending and buckling around a specific reference axis. It is calculated by multiplying each infinitesimal element of area dA by the square of its perpendicular distance from the axis of interest (y or x) and integrating it across the entire section boundary:

$I_{xo} = \int y^2 dA$ and $I_{yo} = \int x^2 dA$

2. Polar Moment of Inertia (J)

The polar moment of inertia measures a cross-section's structural resistance to torsional twisting and rotational deformation when subjected to torque. By the perpendicular axis theorem, the polar moment J about a central longitudinal axis is equal to the direct sum of its rectangular moments of inertia: $J = I_{xo} + I_{yo}$

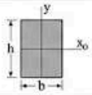
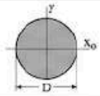
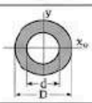
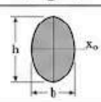
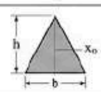
3. Radius of Gyration (k)

The radius of gyration represents a structural property defined as the imaginary radial distance from an axis at which the entire area of a cross-section could be concentrated as a thin strip while yielding the exact same moment of inertia. It is used extensively in column design to evaluate the slenderness ratio and buckling vulnerability: $k = \sqrt{\frac{I_{xo}}{A}}$

4. Section Modulus (Z)

The section modulus is a geometric property calculated by dividing the centroidal area moment of inertia (I_{xo}) by the absolute distance (c) measured from the neutral axis to the outermost, most highly stressed fiber of the cross-section. It directly determines the maximum bending stress experienced by beams ($\sigma = \frac{M}{Z}$): $Z = \frac{I_{xo}}{c}$

Comprehensive Guide to Standard Section Formulas

Shape Profile		Moment of Inertia (I_{xo})	Section Modulus (Z_{xo})	Radius of Gyration (k_{xo})	Polar Moment of Inertia (J)
Rectangle recnangles		$I_{xo} = \frac{bh^3}{12}$	$z_{xo} = \frac{bh^2}{6}$	$k_{xo} = \frac{h}{\sqrt{12}}$	— n/a —
Solid circle		$I_{xo} = \frac{\pi D^4}{64}$	$z_{xo} = \frac{\pi D^3}{32}$	$k_{xo} = \frac{D}{4}$	$J = \frac{\pi D^4}{32}$
Hollow pipe		$I_{xo} = \frac{\pi}{64}(D^4 - d^4)$	$Z_{xo} = \frac{\pi}{32}\left(\frac{D^4 + d^4}{D}\right)$	$k_{xo} = \sqrt{\frac{D^2}{16}}$	$I_{xo} = \frac{\pi bh^3}{64}$ $z_{xo} = \frac{\pi bh^2}{32}$
Solid ellipse		$I_{xo} = \frac{\pi}{64}(D^4 + d^2)$	$Z_{xo} = \frac{\pi}{32}(D^4 - d^4)$	$k_{xo} = \frac{h}{4}$	$J = \frac{\pi}{64}bh(h^2 + b^2)$
Triangle		$I_{xo} = \frac{bh^3}{36}$	—	—	—

Dynamics of Particles

Dynamics is the branch of engineering mechanics that investigates the acceleration, kinematics, and kinetics of particles, rigid bodies, or systems in non-uniform motion. While statics focuses strictly on balanced systems at rest, dynamics addresses the physical parameters that emerge when net external forces disrupt equilibrium. To establish an organized analysis of moving objects, engineering dynamics is divided into two major sub-branches:

- **Kinematics:** This sub-branch explores the purely geometric and temporal aspects of motion. It maps the mathematical relationships between displacement, velocity, acceleration, and time without any regard for the specific forces, torques, or masses that initiate or modify that motion.
- **Kinetics:** This sub-branch bridges the gap between geometry and cause by analyzing the direct relationships between the acting forces, the mass properties of the body, and the resulting non-equilibrium motion it causes.

Fundamental Kinematic Parameters

To mathematically construct the path of a moving particle, engineers track three continuous, time-dependent variables relative to a fixed spatial reference frame:

- **Displacement (s):** This vector quantity represents the net change in position of a particle from an established spatial origin to its current location. Unlike total distance traveled, which measures the complete scalar path taken, displacement depends solely on the initial and final position vectors of the particle.
- **Velocity (v):** This vector parameter defines the instantaneous time rate of change of displacement. It quantifies how fast an object is changing its position relative to a change in time at any specific moment. Mathematically, velocity is defined as the first derivative of displacement with respect to time: $v = \frac{ds}{dt}$

- **Acceleration (a):** This vector parameter defines the instantaneous time rate of change of velocity. It establishes how rapidly the velocity profile is changing at a given instant relative to an infinitesimal progression of time. Mathematically, acceleration is defined as the first derivative of velocity with respect to time: $a = \frac{dv}{dt}$

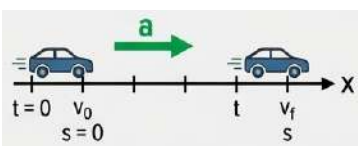
Derivation of the Time-Independent Differential Equation

By analyzing these two absolute derivatives together, the independent time variable (dt) can be algebraically eliminated to establish a direct geometric link between acceleration, velocity, and displacement. Isolating dt in the velocity equation gives $dt = \frac{ds}{v}$. Substituting this expression directly into the acceleration derivative yields:

$$a = \frac{dv}{\left(\frac{ds}{v}\right)} \qquad \Rightarrow \qquad a = v\frac{dv}{ds}$$

Rearranging this relationship results in the fundamental differential equation of kinematics used to solve non-constant acceleration problems: $vdv = ads$

Rectilinear Translation (Straight Line Motion)



Rectilinear translation describes the simplest form of kinematic translation, where a particle moves along a perfectly straight linear path. When a mechanical system experiences a completely uniform, constant acceleration (a = constant), the fundamental differential derivatives can be integrated over a defined time interval from an initial state (t = 0, s = 0, v = v_o) to a final state (t, s, v_f). This integration produces the three standard equations of rectilinear motion:

1. Velocity-Time Relationship

Integrating the acceleration derivative $dv = a\,dt$ over time yields: $v_f = v_o + at$

2. Displacement-Time Relationship

Substituting the velocity-time equation into the displacement derivative $ds = v\,dt$ and integrating yields: $s = v_o t + \frac{1}{2}at^2$

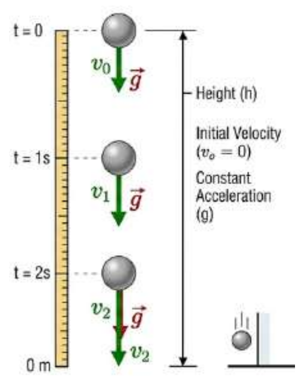
3. Velocity-Displacement Relationship

Integrating the time-independent differential equation $v\,dv = a\,ds$ yields: $v_f^2 = v_o^2 + 2as$

Kinematics of Free Falling Bodies

The motion of a free-falling body represents a specific class of rectilinear translation where an object moves vertically, and its acceleration is governed exclusively by the local gravitational pull of the Earth. In an ideal system where atmospheric air resistance is neglected, all objects accelerate downward at a constant value denoted as g. At sea level, this standard gravitational constant is approximated as g = 9.81 m/s^2 or g = 32.2 ft/s^2.

By substituting the general linear acceleration parameter a with the gravitational constant g, and replacing the horizontal distance variable s with the vertical height variable h, the rectilinear equations transform into the foundational free-fall models:



$v_f = v_o + gt$

$h = v_o t + \frac{1}{2}gt^2$

$v_f^2 = v_o^2 + 2gh$

Projectile Motion

Projectile motion describes a specific class of two-dimensional curvilinear kinematics where a particle is launched into space with an initial velocity vector and moves along a parabolic trajectory under the sole influence of constant gravitational acceleration. In an idealized system where atmospheric air resistance is neglected, the motion is analyzed by breaking it down into two entirely independent, simultaneous kinematic components acting along rectangular coordinate axes: a horizontal component (x-axis) and a vertical component (y-axis).

The initial launch velocity (v_o) at an angle θ relative to the horizontal datum is resolved into its constituent rectangular components: $v_{ox} = v_o cos\theta$ and $v_{oy} = v_o sin\theta$

1. Horizontal Motion Component (x-axis)

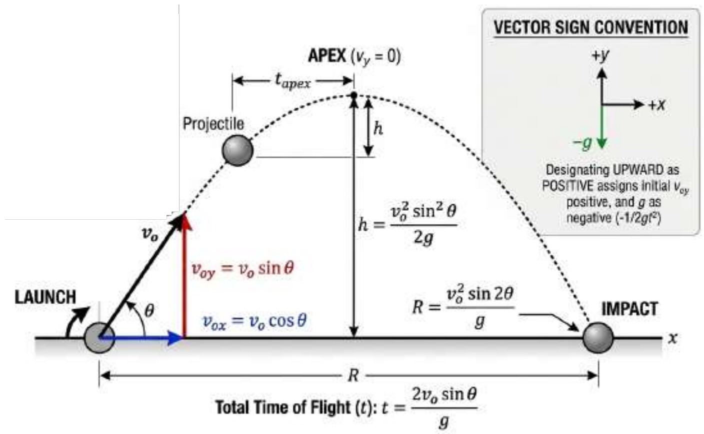
Because there are no external forces acting horizontally on the projectile to alter its speed, the horizontal acceleration is zero ($a_x = 0$). This means the horizontal velocity remains completely uniform and constant throughout the entire flight ($v_x = v_{ox} = v_o cos\theta$). The horizontal displacement x at any given moment of time t is modeled by the constant velocity linear relationship: $x = v_o cos\theta \cdot t$

2. Vertical Motion Component (y-axis)

Conversely, the vertical component is constantly acted upon by the downward pull of gravity $\left(a_y = \pm g\right)$. This means the vertical motion behaves exactly like a free-falling body subjected to constant acceleration. The vertical displacement y at any time interval t is calculated by integrating the free-fall relationship:

$y = v_o sin\theta \cdot t \pm \frac{1}{2}gt^2$

Vector Sign Convention: The choice of the $\pm$ sign in the vertical component depends entirely on your coordinate system. If upward displacement is designated as positive, the initial upward velocity component ($v_o sin\theta$) is positive, while the gravitational acceleration is written as negative ($-\frac{1}{2}gt^2$) because it opposes the upward flight.



Specialized Projectile Formulas

For a projectile launched from a flat ground plane that returns to the exact same elevation at impact, specific equations can be derived to quickly determine key tracking metrics on the board exam:

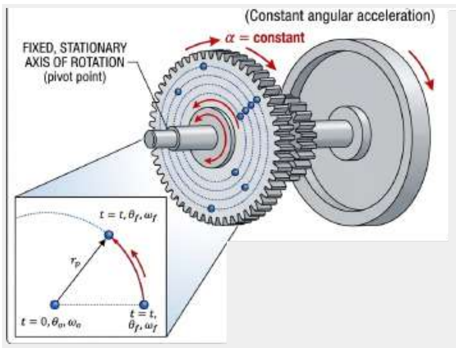
- **Total Time of Flight (t):** At the absolute apex of the parabolic flight path, the instantaneous vertical velocity drops to exactly zero ($v_y = 0$). Using the linear velocity equation $v_y = v_{oy} - gt_{apex}$, the time required to reach peak height is found to be $t_{apex} = \frac{v_o \sin\theta}{g}$. Because the parabolic path is perfectly symmetrical under ideal conditions, the total time of flight is exactly double the time to apex: $t = \frac{2v_o \sin\theta}{g}$

- **Maximum Height (h):** The maximum vertical height h is achieved at the mid-flight apex $\left(t_{apex}\right)$. Substituting this specific time value into the vertical displacement expression yields: $h = \frac{v_o^2 \sin^2\theta}{2g}$

- **Maximum Horizontal Range (R):** The total horizontal range R is the complete distance traveled along the x-axis during the entire flight time t. Substituting the total time of flight into the horizontal displacement equation yields: $R = \frac{v_o^2 \sin 2\theta}{g}$

Rotational Motion of Rigid Bodies

Rotational motion describes a kinematic state where all particles within a rigid body move in concentric circular paths around a fixed, stationary axis. Just as rectilinear motion tracks linear displacement along a straight path, rotational motion tracks angular parameters around a pivot point. When a machine element or component experiences a completely uniform, constant angular acceleration ($\alpha = constant$), the angular equations match the mathematical structure of linear motion equations exactly:



$$\omega_f = \omega_o + \alpha t$$

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega_f^2 = \omega_o^2 + 2\alpha\theta$$

In these fundamental equations, θ represents the total angular displacement measured in **radians** (rad). The parameter ω_o represents the initial angular velocity, while ω_f represents the final angular velocity, both expressed in units of radians per second (rad/sec). The parameter α represents the constant angular acceleration, expressed in units of radians per second squared (rad/sec^2).

Linear and Angular Relationships

To connect the rotational parameters of a rotating component directly to the linear path traveled by an individual point on that component, we use the radius of rotation (r). The radius r represents the perpendicular distance measured from the center axis of rotation out to the specific point of interest.

1. Linear Displacement along a Circular Arc

The linear distance s traveled by a point along its circular arc path is directly proportional to the radius and the total angular displacement it sweeps through. This relationship forms the fundamental definition of a radian: $s = r \cdot \theta$

2. Tangential Linear Velocity

Differentiating the arc length equation with respect to time ($\frac{ds}{dt} = r\frac{d\theta}{dt}$) yields the tangential linear velocity (v). This equation calculates how fast a point on the outer edge of a rotating component (such as a flywheel, gear, or pulley) is moving through space: $v = r \cdot \omega$

When calculating these combined linear-angular relationships, all angular terms **must be expressed in radians**. If an exam problem provides rotational speed in revolutions per minute (rpm), you must always convert it to radians per second (rad/sec) first using the standard conversion factor $\omega = \frac{2\pi N}{60}$ before pairing it with a linear radius dimension.