Thermodynamic Processes

A **thermodynamic process** is defined as the energetic transition of a system from an initial state of equilibrium to a final state of equilibrium. When assessing these transitions for board examinations, they are frequently modeled as **reversible (or quasistatic) processes**. A reversible process is an idealized abstraction wherein the system changes state so infinitesimally slowly that it remains in thermal and mechanical equilibrium with its surroundings at all times. Consequently, the system can be returned to its precise initial state without leaving any net change in either the system or the surroundings, meaning there is zero loss or dissipation of energy.

ISOBARIC PROCESS

Isobaric process occurs at a constant pressure.

For Closed or Non-Flow System

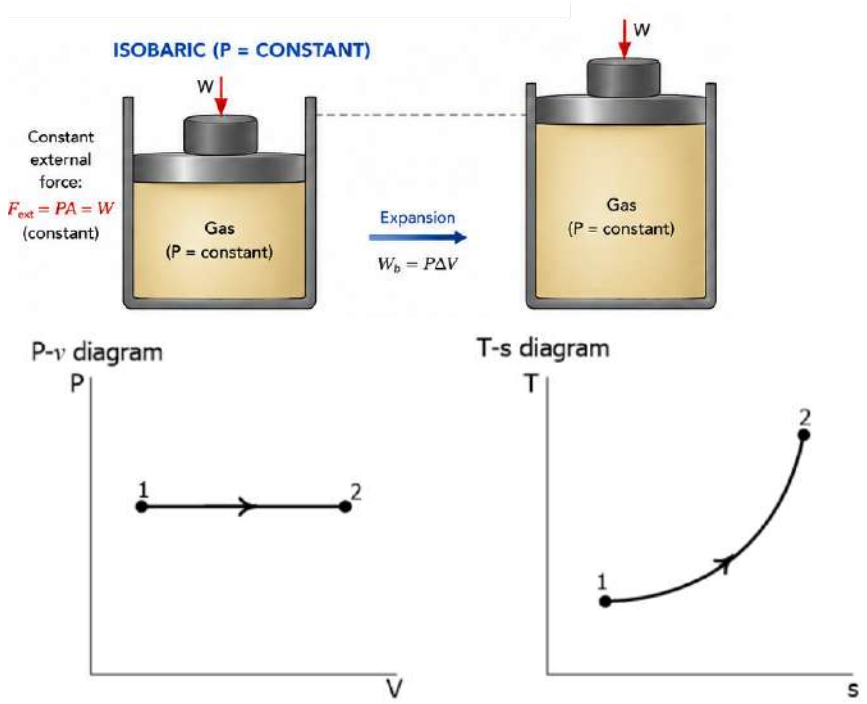


FIGURE 1: Ideal gas confined in a piston-cylinder spring loaded assembly maintaining a constant pressure. Below is the P-V and T-s diagram for the process

Equations:

a. PVT Relationships
For an ideal gas undergoing a change of state, the general characteristic gas equation is expressed as:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = mR = constant$$

Because the pressure remains invariant ($P_1 = P_2$), the pressure terms cancel out entirely. This yields Charles's Law, which states that the volume of an ideal gas is directly proportional to its absolute temperature:

$$\frac{T_2}{T_1} = \frac{V_2}{V_1}$$

b. Work Non-flow, W_{NF}

Thermodynamic non-flow work (or boundary work) is evaluated by integrating the pressure with respect to volume change along the process path:

$$W_{NF} = \int_1^2 P\, dV$$

$$W_{NF} = P\left(V_2 - V_1\right) = P\Delta V$$

Note: $P_2 = P_1 = P$

$$W_{NF} = \left(P_2V_2 - P_1V_1\right) = mR(T_2 - T_1)$$

$$W_{NF} = mR\Delta T$$

c. Change in Total Internal Energy, ΔU

Internal energy is a pure function of temperature for an ideal gas. Regardless of the process path, the change in internal energy depends on the constant-volume specific heat capacity (C_v):

$$\Delta U = mC_v\Delta T$$

d. Change in Total Enthalpy, ΔH

Enthalpy represents the total heat content of a system. By definition, it tracks temperature changes using the constant-pressure specific heat capacity (C_p):

$$\Delta H = mC_p\Delta T$$

e. Heat Energy, Q

To find the heat transferred during a non-flow isobaric process, we apply the Non-Flow Energy Equation (NFEE), which represents the First Law of Thermodynamics for closed systems:

Using NFEE: $Q = \Delta U + W_{NF}$

$$Q = mC_v\Delta T + mR\Delta T$$

$$Q = m\left(C_v + R\right)\Delta T$$

$$Q = mC_p\Delta T = \Delta H$$

f. Change in Total Entropy, ΔS
Entropy measures the microscopic transitions of a system. The fundamental differential relation for entropy change is:

From: $dS = \frac{dQ}{T}$

$$\int_1^2 dS = mC_p \int_1^2 \frac{dT}{T}$$

$$\left(S_2 - S_1\right) = mC_p ln\left(T_2 - T_1\right)$$

$$\Delta S = mC_p ln\frac{T_2}{T_1} = mC_p ln\frac{V_2}{V_1}$$

For Open or Steady-Flow System

In an open or steady-flow system, mass continuously crosses the system boundaries through devices such as heat exchangers, boilers, or condensers operating at constant pressure.

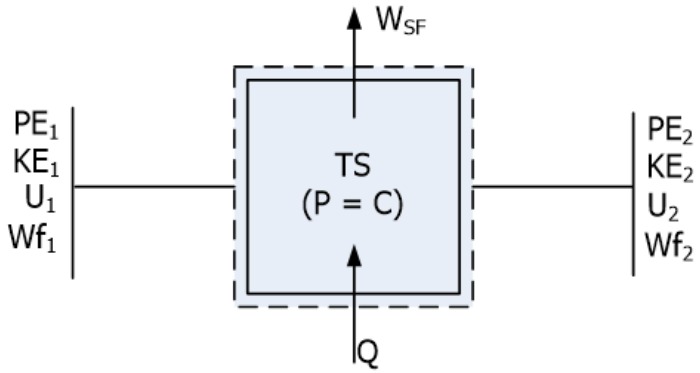


FIGURE 2: Constant pressure process in a steady flow system

To analyze this system, we apply the First Law of Thermodynamics for a steady-flow system, which dictates that the total energy entering the system must equal the total energy exiting ($\left(E_{in} = E_{out}\right)$:

Applying the First Law of Thermodynamics:

$$E_{in} = E_{out}$$

$$PE_1 + KE_1 + U_1 + W_{f1} + Q = W_{SF} + PE_2 + KE_2 + U_2 + W_{f2}$$

$$Q = W_{SF} + \Delta PE + \Delta KE + \Delta U + \Delta W_f$$

When: $z_1 \approx z_2$; $then\ \Delta PE \approx 0$

$v_1 \approx v_2$; $then\ \Delta KE \approx 0$

And: $\Delta H = \Delta U + \Delta W_f$

Therefore, the steady flow energy equation, SFEE is given by:

$Q = W_{SF} + \Delta H$ but: $Q = \Delta H$

$$W_{SF} = 0$$

For an open, steady-flow system operating under purely isobaric conditions with negligible kinetic and potential energy changes, the Steady-Flow Shaft Work (W_{SF}) is completely equal to zero.

Sample Problem:

Nitrogen with a mass of 2.5 kg expands at a constant pressure. The initial pressure and temperature in the piston cylinder system is 170 kPa and 27°C, respectively. Because of the heat transfer, the system reached a final temperature of 200°C. Determine:

(a) The heat required
(b) The work
(c) The change in enthalpy
(d) The change in internal energy

Solution:

For the heat required:

$$Q = mC_p\left(T_2 - T_1\right) = 2.5\ kg\left(1.0399\frac{kJ}{kg-K}\right)(473 - 300)K$$

$$Q = 449.7566\ kJ$$

For the work:

$$W_{NF} = mR\left(T_2 - T_1\right) = 2.5\ kg\left(0.2968\frac{kJ}{kg-K}\right)(473\ -\ 300)K$$

$$W_{NF} = 128.3660\ kJ$$

For the change in enthalpy:

$$\Delta H = mC_p\Delta T = Q$$

$$\Delta H = 449.7566\ kJ$$

For the change in internal energy:

$$\Delta U = mC_v\Delta T = 2.5\ kg\left(0.7431\frac{kJ}{kg-K}\right)(473\ -\ 300)K$$

$$\Delta U = 321.3908\ kJ$$

When you add thermal energy to a gas at constant pressure, not all of it converts into useful mechanical work. In this specific scenario, roughly 71.5% of the heat you paid for via fuel is "trapped" inside the gas as an internal temperature rise (U), while only about 28.5% is converted into useful boundary work (W_{NF}) to move the piston.

HEAT EXCHANGERS

For analytical modeling, an **ideal heat exchanger** is always assumed to operate via an **isobaric process (P=C)**. This means fluid friction, pressure drops due to pipe bends, and internal turbulence are neglected, ensuring that the working fluids enter and leave their respective channels at a completely constant pressure.

Types:

1. Steam generator
    a. Economizer
    b. Steam boiler
    c. Superheater
2. Condenser
3. Evaporator
4. Air pre-heaters
5. Shell and tube heat exchangers
6. Intercoolers
7. Direct contact heat exchangers

Consider a steam boiler shown in Figure 3, assuming that the potential and kinetic energies are negligible.

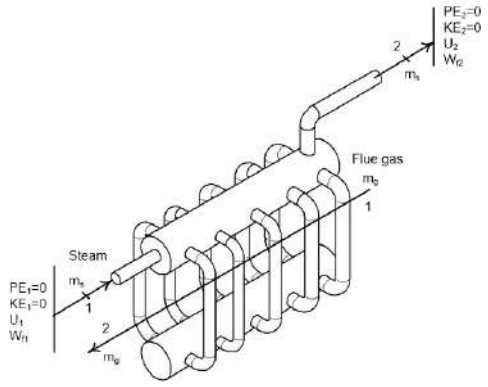


FIGURE 3: Heat transfer from a steam generator

Applying the First Law of Thermodynamics:

$$E_{in} = E_{out}$$

$$H_{g1} + H_{s1} = H_{g2} + H_{s2}$$

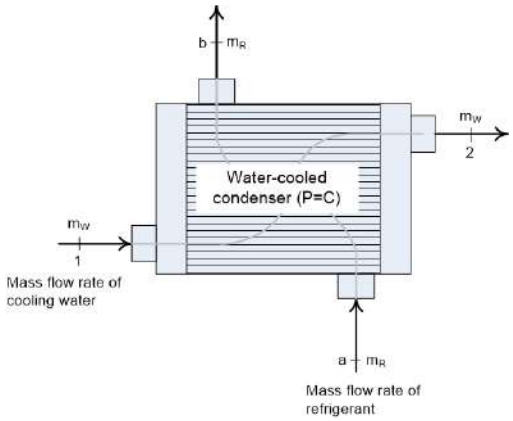
$$H_{g1} - H_{g2} = H_{s2} - H_{s1}$$

$$m_g C_{pg}\left(T_1 - T_2\right)_g = m_s(h_2 - h_1)$$

$$Heat\ energy\ given\ off\ by\ the\ flue\ gas\ =\ heat\ absorbed\ by\ steam$$

Sample Problem:

A refrigerant is flowing through a water-cooled condenser. It is found that the enthalpy of the refrigerant at the entrance and exit of the condenser is 420 kJ/kg and 180 kJ/kg, respectively. If the temperature increase in water is 15°C and the mass flow rate of the refrigerant is 30 kg/min, determine the necessary mass flow rate of cooling water, in kg/min for this application. Assume that the changes in kinetic and potential energies are negligible and there are no external heat losses in the system.



Solution:

Solving for cooling water flow rate:

$$E_{in} = E_{out}$$

$$H_{w1} + H_{Ra} = H_{w2} + H_{Rb}$$

$$H_{w1} - H_{w2} = H_{Rb} - H_{Ra}$$

$$-m_w C_{pw}\left(T_1 - T_2\right)_w = m_R(h_b - h_a)$$

$$-m_w\left(4.187\frac{kJ}{kg-K}\right)(15K) = \left(30\frac{kg}{min}\right)(180 - 420)\frac{kJ}{kg}$$

$$m_w = 114.6404\ \frac{kg}{min}$$

ISOCHORIC PROCESS

An **isochoric process**—alternatively designated in thermodynamic literature as an **isometric** or **isovolumetric process**—is a thermodynamic operation that occurs at a **constant volume ($V_1 = V_2 = V$)**. In a physical system, this state change represents a process where the boundaries are rigid and immovable, preventing any volumetric expansion or compression of the working fluid.

For Closed or Non-Flow System

In a closed or non-flow system, a fixed mass of gas is trapped inside a completely rigid container. Because the vessel walls cannot deform, the gas cannot perform boundary work on its surroundings, nor can the surroundings perform work on the gas.

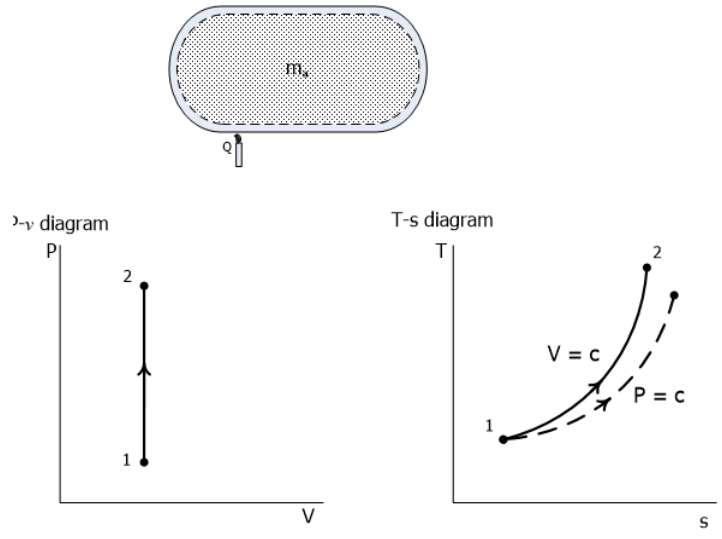


FIGURE 4: Ideal gas confined in a rigid container. Below is the P-V and T-s diagram for the process

Equations:

(a) PVT Relationships

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = mR = constant$$

Because the volume remains constant (**$V_1 = V_2 = V$**), the volumetric terms cancel out completely. This yields Gay-Lussac's Law, which states that the absolute pressure of an ideal gas is directly proportional to its absolute temperature

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$

(b) Work Non-flow, W_{NF}

$$W_{NF} = \int_1^2 P\, dV$$

Because volume is constant (V =constant), the total differential of volume is zero (dV = 0). Substituting dV = 0 into the boundary work integral yields:

$$W_{NF} = 0$$

Therefore, no boundary or non-flow mechanical work is performed during an isochoric process.

(c) Change in Total Internal Energy, ΔU

According to Joule's Law, the internal energy of an ideal gas depends solely on its absolute temperature. The change in internal energy is always calculated using the constant-volume specific heat capacity (C_v), regardless of the process path:

$$\Delta U = mC_v\Delta T$$

(d) Change in Total Enthalpy, ΔH

Enthalpy tracks the total thermal energy and flow energy capability of a fluid. Even though the volume is held constant, the change in enthalpy for an ideal gas must still be evaluated using the constant-pressure specific heat capacity (C_p):

$$\Delta H = mC_p\Delta T$$

(e) Heat Energy, Q

To determine the heat energy transferred during a non-flow isochoric process, we apply the Non-Flow Energy Equation (NFEE), which represents the First Law of Thermodynamics for closed systems:

Using NFEE: $Q = \Delta U + W_{NF}$

$$Q = mC_v \Delta T = \Delta U$$

(f) Change in Total Entropy, ΔS

From: $dS = \frac{dQ}{T}$

$$\int_1^2 dS = mC_v \int_1^2 \frac{dT}{T}$$

$$\left(S_2 - S_1\right) = mC_v ln\left(T_2 - T_1\right)$$

$$\Delta S = mC_v ln \frac{T_2}{T_1} = mC_v ln \frac{P_2}{P_1}$$

For Open or Steady-Flow System

In an open steady-flow system, a fluid passes continuously through a rigid control volume (such as a fixed-volume chamber or a specialized reactor core) where the volumetric flow rate at the inlet equals the volumetric flow rate at the outlet.

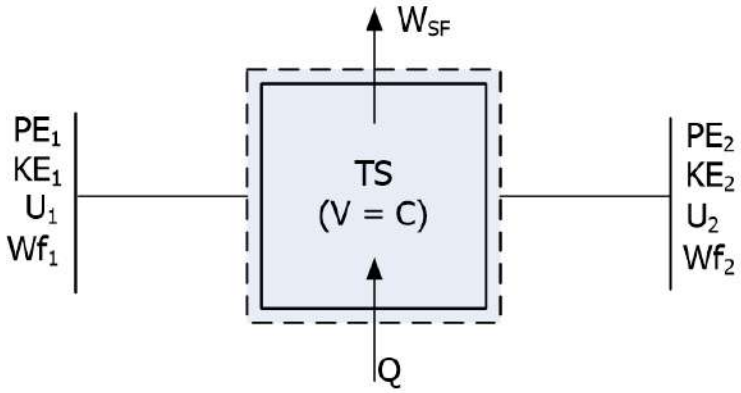


FIGURE 5: Constant volume process in a steady flow system

Applying the First Law of Thermodynamics:

$$E_{in} = E_{out}$$

$$PE_1 + KE_1 + U_1 + W_{f1} + Q = W_{SF} + PE_2 + KE_2 + U_2 + W_{f2}$$

$$Q = W_{SF} + \Delta PE + \Delta KE + \Delta U + \Delta W_f$$

When: $z_1 \approx z_2$; $then\ \Delta PE \approx 0$

$v_1 \approx v_2$; $then\ \Delta KE \approx 0$

$$Q = W_{SF} + \Delta U + \Delta W_f$$

But: $Q = \Delta U$

Therefore, the steady flow energy equation, SFEE is given by:

$$W_{SF} = -\Delta W_f = -\left(P_2V_2 - P_1V_1\right)$$

Since: $V_2 = V_1 = V$

$$W_{SF} = -V\left(P_2 - P_1\right) = -V\Delta P$$

$$W_{SF} = -mR\left(T_2 - T_1\right) = -mR\Delta T$$

Unlike a closed isochoric system (where work is strictly zero), an open, steady-flow isochoric system does perform shaft work. This work is equal to the negative product of the constant volume and the pressure differential (-VΔP). If the pressure increases (ΔP > 0), shaft work is negative, meaning mechanical work must be supplied to the system.

Sample Problem:

A closed rigid container has a volume of 2000 L and holds air initially at 350 kPa and 300 K. Heat is added until the temperature reaches 530 K. Determine the heat added in the system and its final pressure.

Solution:

For the heat added:

$$Q = mC_v \Delta T$$

$$P_1 V_1 = mRT_1$$

$$(350\ kPa)\left(2\ m^3\right) = m\left(0.287\ \frac{kJ}{kg-K}\right)(300K)$$

$$m = 8.13\ kg$$

$$Q = (8.13\ kg)\left(0.7186\ \frac{kJ}{kg-K}\right)(530\ -\ 300)K$$

$$Q = 1,343.7101\ kJ$$

The positive value confirms that 1,343.71kJ of thermal energy must be **transferred into** the container from an external heat source.

For the final pressure:

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$

$$\frac{P_2}{350kPa} = \frac{530\ K}{300K}$$

$$P_2 = 618.3333\ kPa$$

In this scenario, raising the temperature by less than double (300 K to 530 K) caused the internal pressure to climb from a manageable 350 kPa to a significant 618.33 kPa. This behavior explains why compressed air receivers, steam drums, and processing piping must never be isolated without protection. If an external fire breaks out near an unvented, rigid tank, the heat transfer creates an open-ended isochoric pressure rise. This will eventually exceed the mechanical yield strength of the metal walls, leading to a catastrophic pressure vessel rupture. This underscores why mechanical engineers must install properly sized pressure relief valves (PRVs) on all rigid thermal containment systems.

PUMPS

In fluid mechanics and industrial thermodynamics, pumps are essential steady-flow mechanical devices designed to transport liquids from a lower elevation or lower pressure region to a higher elevation or higher pressure region. Structurally, a typical dynamic pump consists of a rotating internal element called an **impeller** housed inside a stationary outer **casing**. The impeller imparts kinetic energy to the liquid, which the casing then converts into pressure energy.

The working fluid handled by a pump (typically water or oil) is treated as an **incompressible fluid**. Because the density (ρ) and specific volume (v) of an incompressible liquid do not change significantly even under immense pressure, the pumping process is modeled thermodynamically as an open system operating via a **constant volume process (V = C)**.

Types:
1. Reciprocating
2. Centrifugal
3. Rotary

Consider a water pump shown in Figure 6, assuming that the potential and kinetic energies are negligible:

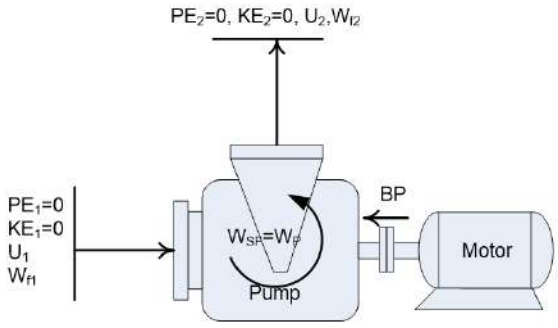


FIGURE 6: Water being compressed by a pump at a constant volume pumping process

From steady-flow general equation: $$W_{SF} = \int_1^2 V\, dP$$

$$W_{SF} = V\left(P_2 - P_1\right)$$

Note: $$\left(P_2 - P_1\right) = \gamma h$$

$$W_{pump} = V\gamma h$$

Note: Always remember that pump work (W_{pump}) is **work done on the system**. In thermodynamic sign conventions, this is treated as a negative work term (W < 0).

Sample Problem:

A pump has a capacity of 200 L/s. If the total dynamic head handled by the pump is 60 m, determine the horsepower output delivered by the pump.

Solution:

$$W_{pump} = -\ V\gamma h$$

$$W_{pump} = -\left(200\frac{L}{s}\right)\left(\frac{1\ m^3}{1000\ L}\right)\left(9.81\ \frac{kN}{m^3}\right)(60m)$$

$$W_{pump} = -\ (117.72\ kW)\left(\frac{1\ hp}{0.746\ kW}\right)$$

$$W_{pump} = \ 158\ hp$$

If this specific pump has an efficiency of 75%, the actual shaft power that must be supplied by an electric motor—known as the **Brake Horsepower (BHP)**—would be significantly higher:

$$BPH\ = \frac{WHP}{\eta_P} = \frac{158\ hp}{0.75} = \ 210.67\ hp$$

Consequently, an engineer would need to specify a standard commercial 225 hp motor to safely run this system without overloading it. Sizing industrial equipment requires always accounting for these real-world losses beyond the theoretical fluid requirements.

SOME EXAMPLES OF ACTUAL EQUIPMENT UNDERGOING ISOMETRIC PROCESS

In thermodynamics, an isometric (or isochoric) process refers to a process that occurs at constant volume, where the system's volume remains unchanged. This type of process typically involves changes in pressure and temperature but no work is done by the system since the volume is fixed. Various types of equipment, such as those used in scientific experiments, gas storage, and fluid systems, can be modeled or simplified as undergoing isometric processes. Below are examples of actual equipment that operate under constant volume or can be approximated as such for certain applications.

Bomb Calorimeter

A bomb calorimeter is used to measure the heat of combustion of a substance under constant volume conditions. The substance is burned in a sealed, rigid container, and the resulting temperature change is used to calculate the heat released.

	**High Pressure Gas Storage Tanks** Gasses stored in rigid, high-pressure cylinders experience constant volume processes when heated or cooled. The pressure changes in the tank, but the volume remains fixed.
	**Pumps** For **pumps**, which typically handle liquids (incompressible fluids), the volume of liquid entering and leaving the pump *may be considered constant*. In such cases, if there is no significant change in the volume of the liquid being pumped, the process can be simplified as approximately constant volume, especially when the density of the fluid remains nearly the same.
	**Fans and Blowers** For **fans** and **blowers**, when air or gas flows at low speeds or under conditions where compressibility effects are negligible, the process might be treated as constant volume. In these cases, the changes in density and volume of air can be small enough that the system behaves similarly to a constant volume process, especially in analyses where flow rate or pressure is the primary concern.

In engineering thermodynamics, real-world processes involving ideal gases are mathematically modeled using a generalized, multiform relationship known as the **polytropic equation**. The word polytropic literally translates to "multiform" or "varied," reflecting its ability to characterize a wide spectrum of distinct thermodynamic paths. The general expression governing any reversible polytropic process, applicable to both closed (non-flow) and open (steady-flow) systems, is written as:

$$P_1 V_1^{\,n} = P_2 V_2^{\,n} = PV^n = C$$

Where: n = polytropic exponent
P = absolute pressure
V = total volume
C = constant

Polytropic means “multiform” or “varied” and this equation applies to reversible processes, both open and closed systems. This equation is also used to describe processes of specific systems as the polytropic exponent varies from zero to infinity.

If n = 0 then P = C: Isobaric Process or Constant Pressure Process
If n = ∞ then V = C: Isochoric Process or Isometric Process or Constant Volume Process
If n = 1 then PV = C: Isothermal Process or Constant Temperature Process
If n = k then PV^k = C: Isentropic Process
If 1 < n < 1.4 (PV^n=C): Polytropic Process

ISOTHERMAL PROCESS

Isothermal processes occur at a constant temperature. An example of this is a system immersed in cooling water maintained at a constant temperature ($T_1 = T_2 = T_3$)

For Closed or Non-Flow System

In a closed or non-flow system, a fixed mass of gas is trapped inside a physical boundary, such as a piston-cylinder assembly. To achieve a perfectly constant temperature during expansion or compression, the system must maintain active thermal contact with an external reservoir. A classic engineering example is a piston-cylinder assembly immersed in a circulating cooling water bath maintained at a constant temperature. As work is performed on or by the gas, heat is rapidly exchanged through the cylinder walls to offset any potential temperature change.

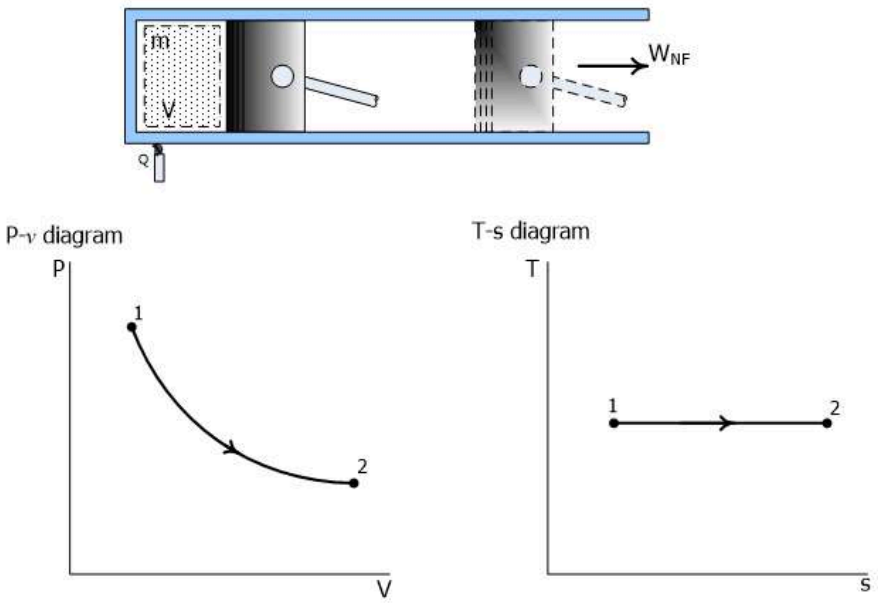


FIGURE 1: Ideal gas confined in a water-cooled piston-cylinder assembly. Below is the P-V and T-s diagram for the process

Equations:

a. PVT Relationships

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = mR = constant$$

Because the temperature is constant ($T_1 = T_2$), the temperature terms cancel completely out of the denominator. This yields Boyle's Law, which demonstrates that absolute pressure and volume share a strictly inverse relationship:

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

b. Work Non-flow, W_{NF}

$$W_{NF} = \int_1^2 P\, dV$$

Where: $P_2V_2 = P_1V_1 = PV = C$

$$P = \frac{C}{V}$$

$$W_{NF} = C\int_1^2 \frac{dV}{V}$$

$$W_{NF} = C\, ln\left(V_2 - V_1\right)$$

$$W_{NF} = C\, ln\, \frac{V_2}{V_1} = C\, ln\, \frac{P_1}{P_2}$$

$$W_{NF} = P_1V_1\, ln\, \frac{V_2}{V_1} = mRT_1\, ln\, \frac{P_1}{P_2}$$

c. Change in Total Internal Energy, ΔU

Because there is zero temperature variation (ΔT = 0) in an isothermal process, both values collapse completely to zero:

$$\Delta U = 0$$

d. Change in Total Enthalpy, ΔH

$$\Delta H = 0$$

e. Heat Energy, Q

To determine the thermal energy exchange, we apply the Non-Flow Energy Equation (NFEE), representing the First Law of Thermodynamics for closed systems:

Using NFEE: $Q = \Delta U + W_{NF}$

$$Q = W_{NF}$$

In a closed, reversible isothermal process, the thermal energy added to the system is converted entirely into boundary work output, leaving the system's internal energy completely unchanged.

f. Change in Total Entropy, ΔS

From: $dS = \frac{dQ}{T}$

$$\int_1^2 dS = \frac{1}{T}\int_1^2 dQ = \frac{Q}{T}$$

$$\left(S_2 - S_1\right) = \frac{mRT_1 ln\frac{P_1}{P_2}}{T}$$

$$\Delta S = mR\, ln\, \frac{P_1}{P_2} = mR\, ln\, \frac{V_2}{V_1}$$

For Open or Steady-Flow System

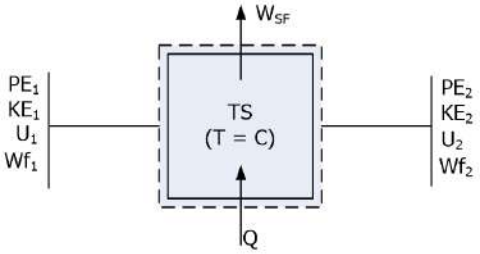


FIGURE 2: Constant temperature process in a steady flow system

Applying the First Law of Thermodynamics:

$$E_{in} = E_{out}$$

$$PE_1 + KE_1 + U_1 + W_{f1} + Q = W_{SF} + PE_2 + KE_2 + U_2 + W_{f2}$$

$$Q = W_{SF} + \Delta PE + \Delta KE + \Delta U + \Delta W_f$$

When: $z_1 \approx z_2\,;\ then\ \Delta PE \approx 0$

$v_1 \approx v_2\,;\ then\ \Delta KE \approx 0$

And: $\Delta H = \Delta U + \Delta W_f$

Therefore, the steady flow energy equation, SFEE is given by:

$$Q = W_{SF} + \Delta H \qquad \text{but:} \quad \Delta H = 0$$

$$W_{SF} = Q$$

For an ideal gas undergoing a steady-flow isothermal process with negligible kinetic and potential energy variations, the useful **Steady-Flow Shaft Work (W_{SF})** is exactly equal to the **Heat Transferred (Q)**, which perfectly mirrors the magnitude of **Non-Flow Work (W_{NF})**.

Sample Problem:

Using a large water cooled compressor, air at 8.4 kg/min is compressed at a constant temperature from an initial temperature and pressure of 27°C and 100 kPa, respectively, to a final pressure of 550 kPa. Neglecting changes in Potential and Kinetic Energy. Determine:

(a) Entropy change
(b) Power required

Solution:

$$\Delta S = mR \ln ln \frac{P_1}{P_2} = \left(\frac{8.4\ kg}{60\ s}\right)\left(0.287\ \frac{kJ}{kg-K}\right) \ln ln \left(\frac{100\ kPa}{550\ kPa}\right)$$

$$\Delta S = -\ 0.06850 \frac{kJ}{s-K}$$

$$W_{SF} = mR \ln T_1 ln \frac{P_1}{P_2} = \left(\frac{8.4\ kg}{60\ s}\right)\left(0.287\ \frac{kJ}{kg-K}\right)(300\ K) \ln ln \left(\frac{100\ kPa}{550\ kPa}\right)$$

$$W_{SF} = -\ 20.5490\ kW$$

By actively circulating cooling water around the compressor cylinders, we can force the air to undergo an approximation of an **isothermal process**. Because the temperature is kept low, the air remains dense, minimizing the volume the piston has to push against. This reduces the area under the steady-flow curve, making an isothermal compression path the most energy-efficient baseline available.

ISENTROPIC PROCESS

In engineering thermodynamics, an **isentropic process** is defined as a thermodynamic change of state that is both **reversible** (quasistatic and frictionless) and **adiabatic** (perfectly heat-insulated). The term adiabatic dictates that no heat energy is added to or removed from the system ($Q = 0$). For this to happen theoretically, the system must be entirely isolated from its surroundings by a perfectly insulated boundary.

When a process is simultaneously frictionless (reversible) and adiabatic, the total entropy of the system remains perfectly constant ($S_1 = S_2$). Therefore, an isentropic process is fundamentally a **constant-entropy process** ($\Delta S = 0$). On a standard polytropic index scale where $PV^n = C$, the isentropic process occurs when the polytropic exponent n equals the specific heat ratio k (where $k = C_p/C_v$).

For Closed or Non-Flow System

In a closed or non-flow system, a fixed mass of an ideal gas is trapped inside a physical boundary, typically modeled as a piston-cylinder assembly wrapped in perfect thermal insulation.

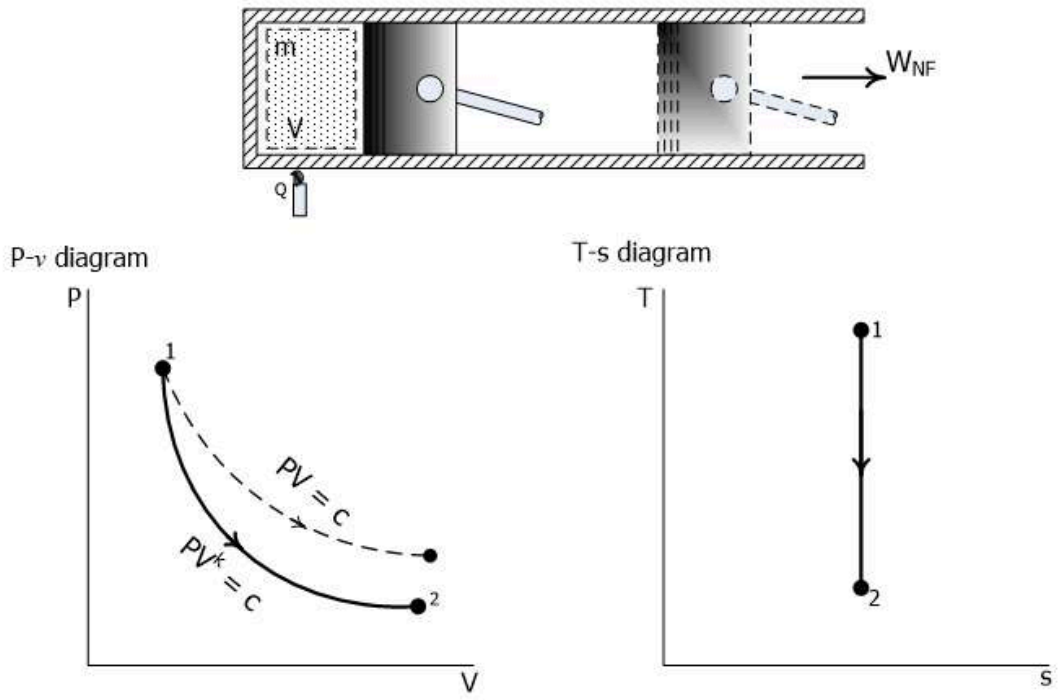


FIGURE 3: Ideal gas confined in a perfectly insulated piston-cylinder assembly. Below is the P-V and T-s diagram for the process

Equations:

(a) PVT Relationships

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

(b) Work Non-flow, W_{NF}

$$W_{NF} = \int_1^2 P\, dV$$

Where: $P_2V_2^k = P_1V_1^k = PV^k = C$

$$P = \frac{C}{V^k} = CV^{-k}$$

$$W_{NF} = C\int_1^2 V^{-k} dV$$

$$W_{NF} = C\left[\frac{V^{-k+1}}{-k+1}\right]_1^2$$

$$W_{NF} = \left[\frac{P_2V_2^kV_2^{-k+1} - P_1V_1^kV_1^{-k+1}}{-k+1}\right]$$

$$W_{NF} = \frac{P_2V_2 - P_1V_1}{1-k} = \frac{mR(T_2 - T_1)}{1-k}$$

(c) Change in Total Internal Energy, ΔU

$$\Delta U = mC_v\Delta T$$

(d) Change in Total Enthalpy, ΔH

$$\Delta H = mC_p\Delta T$$

(e) Heat Energy, Q

$$Q = T\int_1^2 dS$$

The integral of a constant entropy boundary (dS = 0) yields zero net heat transfer."

$$Q = 0$$

Using NFEE: $Q = \Delta U + W_{NF}$

$$W_{NF} = -\Delta U$$

g. Change in Total Entropy, ΔS

$$\Delta S = 0$$

In a closed isentropic expansion, the mechanical work output is produced entirely by sacrificing the system's internal energy, causing a drop in temperature. In an isentropic compression, the work input goes directly into raising the gas's internal energy, causing a sharp temperature spike.

For Open or Steady-Flow System

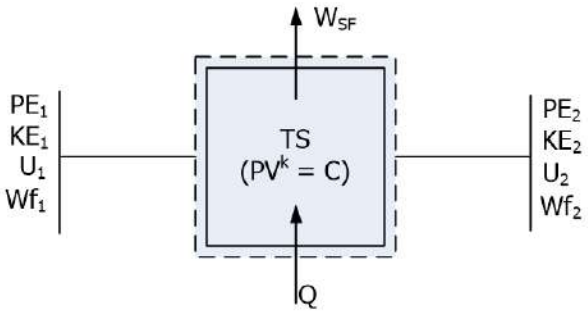


FIGURE 4: Isentropic or Constant Entropy process in a steady flow system

Applying the First Law of Thermodynamics:

$$E_{in} = E_{out}$$

$$PE_1 + KE_1 + U_1 + W_{f1} + Q = W_{SF} + PE_2 + KE_2 + U_2 + W_{f2}$$

$$Q = W_{SF} + \Delta PE + \Delta KE + \Delta U + \Delta W_f$$

When: $z_1 \approx z_2$; $then\ \Delta PE \approx 0$

$v_1 \approx v_2$; $then\ \Delta KE \approx 0$

And: $\Delta H = \Delta U + \Delta W_f$

Therefore, the steady flow energy equation, SFEE is given by:

$Q = W_{SF} + \Delta H$ but: $Q = 0$

$$W_{SF} = -\Delta H$$

Sample Problem

Air in a piston-cylinder occupies an initial volume of 0.15m^3 at a pressure of 560 kPa. The air expands in a reversible adiabatic process. Air doing the work on the piston causes the volume to increase until it reaches 0.30 m^3. Determine the work in the system.

Solution:

$$W_{NF} = \frac{P_2V_2 - P_1V_1}{1-k}$$

$$\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

$$\left(\frac{P_2}{560\ kPa}\right)^{\frac{1.4-1}{1.4}} = \left(\frac{0.15\ m^3}{0.30\ m^3}\right)^{1.4-1}$$

$$P_2 = 212.2003\ kPa$$

$$W_{NF} = \frac{(212.2003\ kPa)(0.30\ m^3) - (560\ kPa)(0.30\ m^3)}{1-1.4}$$

$$W_{NF} = 50.8498\ kJ$$

An adiabatic expansion represents a system working entirely on its own "stored reserves." Unlike an isothermal expansion (where an external heat source continuously helps the gas push the piston), an adiabatic gas runs out of energy faster because its internal pressure and temperature crash simultaneously. As a power plant engineer, this demonstrates why reheating stages are required between steam or gas turbine sections to maximize industrial power cycles.

POLYTROPIC PROCESS

In engineering thermodynamics, actual physical processes seldom fit perfectly into the idealized frameworks of isobaric, isochoric, isothermal, or isentropic paths. Instead, real-world compression and expansion processes involving ideal gases experience simultaneous variations in pressure, volume, temperature, and heat transfer. To model these transitions, engineers utilize the generalized **polytropic process**, which obeys the fundamental thermodynamic relation: $P_1 V_1^{\ n} = P_2 V_2^{\ n} = PV^n = C$

Typically, for real-world engines, compressors, and gas turbines operating under normal atmospheric constraints, this exponent falls within the range of 1 < n < k, where k is the specific heat ratio (C_p/C_v).

For Closed or Non-Flow System

This behavior is structurally modeled using components such as an air-cooled piston-cylinder assembly, where the heat transferred through the cylinder walls prevents the system from being perfectly adiabatic (n=k), yet does not cool it fast enough to remain perfectly isothermal (n=1).

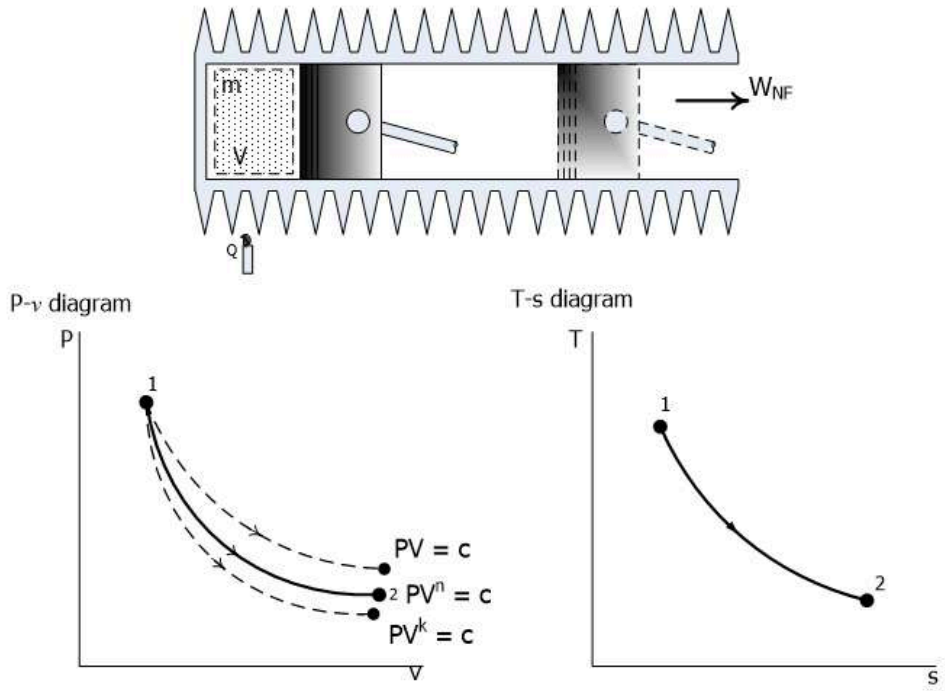


FIGURE 5: Ideal gas confined in an air-cooled piston-cylinder assembly. Below is the P-V and T-s diagram for the process

Equations:

(a) PVT Relationships

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

Derivation:

$$P_1 V_1^{\,n} = P_2 V_2^{\,n}$$

$$\left[\frac{mRT_1}{V_1}\right] V_1^{\,n} = \left[\frac{mRT_2}{V_2}\right] V_2^{\,n}$$

$$T_1 V_1^{\,n-1} = T_2 V_2^{\,n-1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

And:

$$P_1 V_1^{\,n} = P_2 V_2^{\,n}$$

$$P_1\left[\frac{mRT_1}{P_1}\right]^n = P_2\left[\frac{mRT_2}{P_2}\right]^n$$

$$P_1\left(\frac{T_1^{\,n}}{P_1^{\,n}}\right) = P_2\left(\frac{T_2^{\,n}}{P_2^{\,n}}\right)$$

$$\frac{T_1^{\,n}}{P_1^{\,n-1}} = \frac{T_2^{\,n}}{P_2^{\,n-1}}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

(b) Work Non-flow, W_{NF}

$$W_{NF} = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n}$$

(c) Heat Energy, Q

Using NFEE: $Q = \Delta U + W_{NF}$

$$Q = mC_v\Delta T + \frac{mR\Delta T}{1-n}$$

$$Q = \frac{(1-n)mC_v\Delta T + mR\Delta T}{1-n}$$

$$Q = \frac{mC_v\Delta T - nmC_v\Delta T + mR\Delta T}{1-n}$$

$$Q = \frac{m\left(C_v + R\right)\Delta T - nmC_v\Delta T}{1-n}$$

$$Q = \frac{mC_p\Delta T - nmC_v\Delta T}{1-n}$$

Where: $C_p = kC_v$

$$Q = \frac{kmC_v\Delta T - nmC_v\Delta T}{1-n}$$

$$Q = mC_v\left(\frac{k-n}{1-n}\right)\Delta T$$

Let: $C_n = C_v\left(\frac{k-n}{1-n}\right)$

$$Q = mC_n\Delta T$$

Because n is typically greater than 1 in real-world expansion and compression processes (1 < n < k), the denominator term (1 - n) becomes negative, making C_n a **negative value**. A negative specific heat indicates a simultaneous transfer of heat and work. For example, during a polytropic compression where $1 < n < k, heat must be actively rejected (Q is negative) even though the temperature rises (ΔT is positive).

(d) Change in Total Internal Energy, ΔU

$$\Delta U = mC_v\Delta T$$

(e) Change in Total Enthalpy ΔH

$$\Delta H = mC_p\Delta T$$

(f) Change in Total Entropy, ΔS

From: $dS = \frac{dQ}{T}$

$$\int_1^2 dS = mC_n \int_1^2 \frac{dT}{T}$$

$$\Delta S = \left(S_2 - S_1\right) = mC_n ln \frac{T_2}{T_1}$$

$$\Delta S = mC_n ln \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$\Delta S = mC_n ln \left(\frac{V_1}{V_2}\right)^{n-1}$$

For Open or Steady-Flow System

In an open steady-flow system, a gas passes continuously through an uninsulated, moving component (such as an air-cooled industrial compressor or a gas turbine expanding heat into its surroundings).

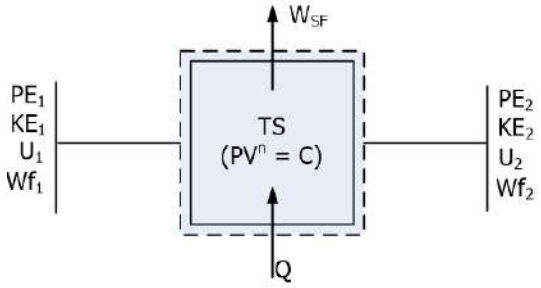


FIGURE 6: Polytropic process in a steady flow system

Applying the First Law of Thermodynamics:

$$E_{in} = E_{out}$$

$$PE_1 + KE_1 + U_1 + W_{f1} + Q = W_{SF} + PE_2 + KE_2 + U_2 + W_{f2}$$

$$Q = W_{SF} + \Delta PE + \Delta KE + \Delta U + \Delta W_f$$

When: $z_1 \approx z_2$; $then\ \Delta PE \approx 0$

$v_1 \approx v_2$; $then\ \Delta KE \approx 0$

And: $\Delta H = \Delta U + \Delta W_f$

Therefore, the steady flow energy equation, SFEE is given by:

$$Q = W_{SF} + \Delta H$$

$$W_{SF} = Q - \Delta H$$

Sample Problem

Air is polytropically compressed in a cylinder according to $PV^{1.3}$=C. The work required is 190 kJ. Determine:

(a) The change in internal energy

(b) The heat transferred

Solution:

For the change in internal energy,

$$\Delta U = mC_v \Delta T$$

$$W_{NF} = \frac{mR\,\Delta T}{1-n}$$

$$\Delta T = \frac{W_{NF}(1-n)}{mR}$$

$$\Delta U = mC_v\left[\frac{W_{NF}(1-n)}{mR}\right] = \left(0.7186\ \frac{kJ}{kg-K}\right)\left[\frac{(-190\ kJ)(1-1.3)}{0.287\ \frac{kJ}{kg-K}}\right]$$

$$\Delta U = 142.7185\ kJ$$

The positive sign indicates that the internal energy of the air **increased**. The mechanical energy crushed into the cylinder directly elevated the gas's molecular kinetic energy, causing its temperature to rise.

For the heat transferred,

$$Q = mC_n \Delta T$$

$$C_n = C_v\left(\frac{k-n}{1-n}\right) = \left(0.7186\ \frac{kJ}{kg-K}\right)\left(\frac{1.4-1.3}{1-1.3}\right) = -\ 0.2395\ \frac{kJ}{kg-K}$$

$$Q = mC_n\left[\frac{W_{NF}(1-n)}{mR}\right] = \left(-\ 0.2395\ \frac{kJ}{kg-K}\right)\left[\frac{(-190\ kJ)(1-1.3)}{0.287\ \frac{kJ}{kg-K}}\right]$$

$$Q = -\ 47.5662\ kJ$$

When 1 < n < k (here, 1 < 1.3 < 1.4), the process describes a compressor that has *imperfect cooling*. Out of the 190kJ of electrical/mechanical energy you paid for to run this compressor, **75.1%** stayed trapped in the gas as an internal energy temperature rise (ΔU), while **24.9%** was successfully pushed out through the cylinder walls as rejected heat (Q).

GAS COMPRESSORS

Gas compressors are devices that increase the pressure of a gas by lowering its volume. Compressed gasses are widely used in various applications such in transporting gas from production site to the consumer, for all types of pneumatic tools, also in jet engines to provide air for combustion of fuel and many more.

Types: 1. Centrifugal
2. Axial-flow
3. Reciprocating
4. Rotary screw
5. Scroll

Air compressor basic components are shown in Figure 7:

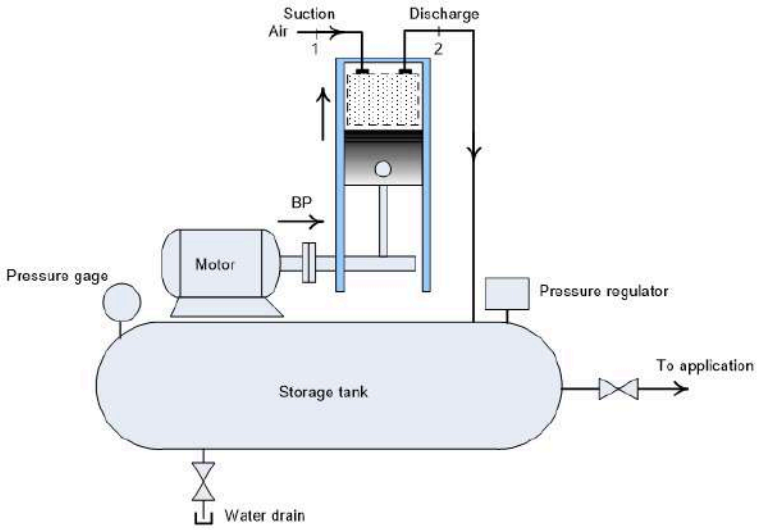


FIGURE 7: Air compressor components

Compressors function as open, steady-flow thermodynamic systems. The fundamental mechanical shaft work required by any open steady-flow device, assuming negligible changes in kinetic and potential energies, is derived using the standard pressure-volume flow integral:

$$W_{SF} = -\int_{1}^{2} V dP$$

In compressor analysis, the subscript "s" denotes the *suction* state (inlet/low pressure), while the subscript "d" signifies the *discharge* state (outlet/high pressure). Depending on how efficiently heat is removed from the system during the process, compression work follows one of three distinct thermodynamic paths.

THEORETICAL WORK OF COMPRESSION

(a) Isothermal compression (T=C)

$$W_C = P_s \ln V_s ln \frac{P_s}{P_d}$$

(b) Isentropic compression ($PV^k=C$)

General Formula for Work steady-flow: $W_{SF} = -\int_1^2 V\, dP$

Where: $PV^k = P_1 V_1{}^k = P_2 V_2{}^k = C$

$$V^k = \frac{C}{P}$$

$$V = \frac{C^{\frac{1}{k}}}{P^{\frac{1}{k}}} = C^{\frac{1}{k}} \bullet P^{-\frac{1}{k}}$$

$$W_{SF} = -C^{\frac{1}{k}} \int_1^2 P^{-\frac{1}{k}}\, dP$$

$$W_{SF} = -C^{\frac{1}{k}} \left[\frac{P^{-\frac{1}{k}+1}}{\left(-\frac{1}{k}+1\right)} \right]_1^2 = -C^{\frac{1}{k}} \left[\frac{P_2{}^{-\frac{1}{k}+1} - P_1{}^{-\frac{1}{k}+1}}{\frac{k-1}{k}} \right]$$

$$W_{SF} = -P^{\frac{1}{k}} V \left[\frac{P_2{}^{-\frac{1}{k}+1} - P_1{}^{-\frac{1}{k}+1}}{\frac{k-1}{k}} \right]$$

$$W_{SF} = k\left[\frac{P_2 V_2 - P_1 V_1}{k-1} \right] = k\left[\frac{mR(T_2 - T_1)}{k-1} \right]$$

$$W_C = k\left[\frac{P_d V_d - P_s V_s}{k-1} \right]$$

(c) Polytropic compression ($PV^n=C$)

$$W_C = n\left[\frac{P_d V_d - P_s V_s}{n-1} \right]$$