Answer Key

Thermodynamic Cycles

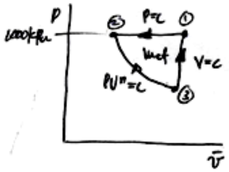
6

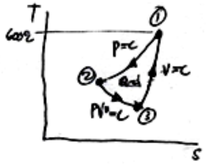
$m = 3\ kg$ of air

process 1-2: constant pressure heat rejection

process 2-3: polytropic expansion $PV^{1.3}$

process 3-1: const volume heat addition





$Q_{1-2} = -200\ kJ$ (heat rejected @ P=c)

a) Pressure and temperature at the end of polytropic expansion, P_3 & T_3

process 3-1:

$$\frac{P_3}{P_1} = \frac{T_3}{T_1}$$

for T_3; process 2-3:

$$\frac{T_3}{T_2} = \left(\frac{V_2}{V_3}\right)^{n-1}$$

$$Q_{1-2} = m\, c_p\, (T_2 - T_1)$$

$$-200\ kJ = (3\ kg)(1.0062\ kJ/kg\text{-}K)(T_2 - 873)K$$

$$T_2 = 806.7441\ K$$

For $V_3 = V_1$

$$P_1 V_1 = m R T_1$$

$$(1000\ kPa)(V_1) = 3\ kg\,(0.287\ kJ/kg\text{-}K)(873\ K)$$

$$V_1 = V_3 = 0.7517\ m^3$$

For V_2; $P_2 V_2 = m R T_2$

$$(1000\ kPa)(V_2) = 3\ kg\,(0.287\ kJ/kg\text{-}K)(806.7441\ K)$$

$$V_2 = 0.6946\ m^3$$

Hence;

$$\frac{T_3}{806.7441\ K} = \left(\frac{0.6946\ m^3}{0.7517\ m^3}\right)^{1.3-1}$$

$$T_3 = 787.8510\ K$$

Then;

$$\frac{P_3}{1000\ kPa} = \frac{787.8570\ K}{873\ K}$$

$$P_3 = 902.4639\ kPa$$

b) The heat added, Q_A

process 2-3: $Q_{2-3} = m\, C_n\, (T_3 - T_2)$

$$C_n = \left[C_v\left(\frac{k-n}{1-n}\right)\right] = 0.7186\ kJ/kg\text{-}K\left(\frac{1.4-1.3}{1-1.3}\right)$$

$$C_n = -0.2395\ kJ/kg\text{-}K$$

$$Q_{2-3} = (3\ kg)(-0.2395\ kJ/kg\text{-}K)(787.8510 - 806.7441)K$$

$$Q_{2-3} = 13.5797\ kJ$$

process 3-1: $Q_{3-1} = m\, C_v\, (T_1 - T_3)$

$$= (3\ kg)(0.7186\ kJ/kg\text{-}K)(873 - 787.8510)K$$

$$Q_{3-1} = 183.5692\ kJ$$

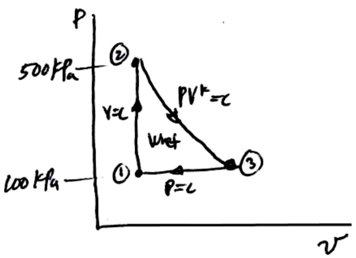
$$Q_A = \Sigma +Q = (13.5797 + 183.5642)\ kJ$$

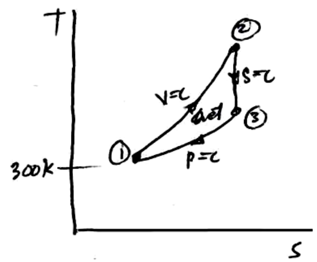
$$\boxed{Q_A = 197.1389\ kJ}$$

7 to 8

$m = 2\ kg/s$

process 1-2: constant volume heat addition
process 2-3: isentropic expansion
process 3-1: constant pressure heat rejection.





a) Heat transfer for each process

process 1-2: $Q_{1-2} = m C_v (T_2 - T_1)$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}$$

$$\frac{T_2}{300k} = \frac{500\ kPa}{100\ kPa}$$

$$T_2 = 1500\ k$$

$$Q_{1-2} = (2\ kg/s)(0.7186\ kJ/kg\text{-}k)(1500 - 300)\ k$$

$$Q_{1-2} = 1724.64\ kJ/s$$

process 2-3: $Q_{2-3} = 0$

process 3-1: $Q_{3-1} = m C_p (T_1 - T_3)$

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{k-1}{k}}$$

$$\frac{T_3}{1500\ k} = \left(\frac{100\ kPa}{500\ kPa}\right)^{\frac{1.4-1}{1.4}}$$

$$T_3 = 947.0776\ k$$

$$Q_{3-1} = (2\ kg/s)(1.0062\ kJ/kg\text{-}k)(300 - 947.0776)\ k$$

$$Q_{3-1} = -1302.1790\ kJ/s$$

b) The work net output;

$$W_{net} = Q_A - Q_R$$

$$= (1724.64 - 1302.1790)\ kJ/s$$

$$\boxed{W_{NET} = 422.4610\ kW}$$

c) Thermal efficiency; $e_c = \frac{W_{NET}}{Q_A} * 100\% = \frac{422.4610\ kJ/s}{1724.64\ kJ/s} * 100\%$

$$\boxed{e_c = 24.50\ \%}$$

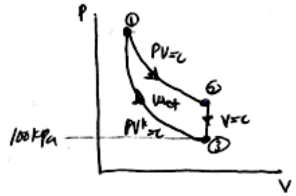
9 to 10

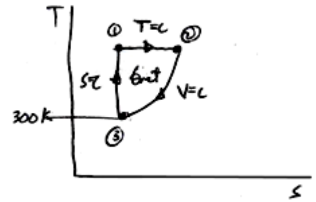
$m = 0.5$ kg of air

process 1-2: Isothermal Expansion

process 2-3: Constant Volume

process 3-1: Isentropic





r_e = expansion ratio $= \dfrac{V_2}{V_1} = \dfrac{V_3}{V_1} = 6.0$

(a) P, V, and T for each state points of the cycle.

$$P_3 V_3 = mRT_3$$

$$(100\ \text{kPa})V_3 = (0.5\ \text{kg})(0.287\ \text{kJ/kg-K})(300\ \text{K})$$

$$V_3 = 0.4305\ \text{m}^3 = V_2$$

process 3-1: $\left(\dfrac{P_1}{P_3}\right)^{\frac{k-1}{k}} = \left(\dfrac{V_3}{V_1}\right)^{k-1}$

$$\left(\frac{P_1}{100\ \text{kPa}}\right)^{\frac{1.4-1}{1.4}} = (6.0)^{1.4-1}$$

$$P_1 = 1228.6035\ \text{kPa}$$

$$\frac{T_1}{T_3} = \left(\frac{V_3}{V_1}\right)^{k-1}$$

$$\frac{T_1}{300\ \text{K}} = (6.0)^{1.4-1}$$

$$T_1 = 614.3017\ \text{K} = T_2$$

from $r_e = \dfrac{V_2}{V_1}$

$$6.0 = \frac{0.4305\ \text{m}^3}{V_1}$$

$$V_1 = 0.07175\ \text{m}^3$$

(b) Q_{in} & Q_{out};

process 1-2: Isothermal heat addition

$$Q_{1-2} = P_1V_1 \ln\frac{P_1}{P_2} = mRT_1 \ln\frac{V_2}{V_1}$$

$$= (0.5\ \text{kg})(0.287\ \text{kJ/kg-K})(614.3017\ \text{K})\ln(6.0)$$

$$Q_{1-2} = 157.9477\ \text{kJ}$$

process 2-3: Isochoric heat rejection

$$Q_{2-3} = mC_v(T_3 - T_2)$$

$$= (0.5\ \text{kg})(0.7186\ \text{kJ/kg-K})(300 - 614.3017)\ \text{K}$$

$$Q_{2-3} = -112.9286\ \text{kJ}$$

(c) $W_{net} = Q_A - Q_R$

$$W_{net} = (157.9477 - 112.9286)\ \text{kJ}$$

$$\boxed{W_{net} = 45.0191\ \text{kJ}}$$

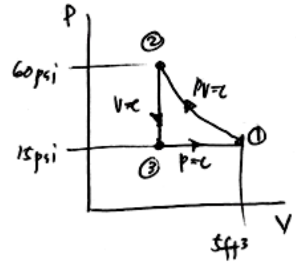
(d) $e_c = \dfrac{W_{net}}{Q_A} * 100\% = \dfrac{45.0191\ \text{kJ}}{157.9477\ \text{kJ}} * 100\%$

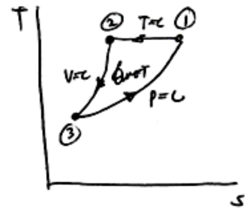
$$\boxed{e_c = 28.50\%}$$

process 1-2: Isothermal compression
from $P_1 = 15\,psi$ to $P_2 = 60\,psi$
$V_1 = 5\,ft^3$

process 2-3: Constant volume

process 3-1: Constant pressure





$W_{net} = \sum W_{NF}$

Process 1-2: $W_{NF_{1-2}} = Q_{1-2} = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right) = P_1 V_1 \ln\left(\frac{P_1}{P_2}\right)$

$= \left(15\frac{lb}{in^2}\right)\left(\frac{12\,in}{1\,ft}\right)^2 (5\,ft^3) \ln\left(\frac{15\,psi}{60\,psi}\right)$

$W_{NF_{1-2}} = -(14971.9791\ lb_f\text{-}ft)\left(\frac{1\,Btu}{778\ ft\text{-}lb_f}\right)$

$W_{NF_{1-2}} = -19.2442\ Btu$

process 2-3: $W_{NF_{2-3}} = 0$

process 3-1: $W_{NF_{3-1}} = P_1 V_1 - P_3 V_3$

$= \left(15\frac{lb}{in^2}\right)\left(\frac{12\ in}{1\,ft}\right)^2 [5 - 1.25]\,ft^3$

$W_{NF_{3-1}} = (8100\ lb_f\text{-}ft)\left(\frac{1\,Btu}{778\ ft\text{-}lb_f}\right)$

$W_{NF\,3-1} = 10.4113\ Btu$

Hence, $W_{NET} = (-19.2442 + 10.4113)\ Btu$

$\boxed{Q_{NET} = W_{NET} = -8.8329\ Btu}$ (work done on the system)