

Step 1: Establish the Intermediate State Absolute Temperatures

Temperature at State 2 (T_2):

Using the isentropic pressure-temperature relation, the temperature at the end of the compression stroke is calculated as follows:

$$T_2 = T_1 \left[\frac{P_2}{P_1} \right]^{\frac{k-1}{k}} = (300 \text{ K}) \left[\frac{1200 \text{ kPa}}{150 \text{ kPa}} \right]^{\frac{1.4-1}{1.4}} = 543.4342 \text{ K}$$

Temperature at State 3 (T_3):

Process 2-3 describes an isobaric expansion, meaning $P_3 = P_2 = 1200 \text{ kPa}$. Because the cycle resets from State 3 back to State 1 through a constant volume (isometric) process, the volume at State 3 is equal to the volume at State 1 ($V_3 = V_1$). Applying Charles' Law for a constant-volume state change between points 3 and 1 gives:

$$T_3 = T_1 \left[\frac{P_3}{P_1} \right] = (300 \text{ K}) \left[\frac{1200 \text{ kPa}}{150 \text{ kPa}} \right] = 2400 \text{ K}$$

Step 2: Compute Cycle Heat Flows

- **Process 1-2 (Isentropic):** No heat transfer occurs along a perfectly insulated adiabatic path:

$$\frac{Q_{1-2}}{m} = 0$$

- **Process 2-3 (Isobaric Heat Addition):** Heat added at constant pressure is calculated using the change in enthalpy:

$$\frac{Q_{2-3}}{m} = C_p (T_3 - T_2) = \left(1.0062 \frac{\text{kJ}}{\text{kg-K}} \right) (2400 - 543.4342) \text{ K} = 1868.0765 \frac{\text{kJ}}{\text{kg}}$$

- **Process 3-1 (Isometric Heat Rejection):** Heat rejected at constant volume is evaluated through the internal energy change:

$$\frac{Q_{3-1}}{m} = C_v (T_1 - T_3) = \left(0.7186 \frac{\text{kJ}}{\text{kg-K}} \right) (300 - 2400) \text{ K} = -1509.0600 \frac{\text{kJ}}{\text{kg}}$$

Step 3: Compute Cycle Work

- **Process 1-2: Isentropic Compression (W_{1-2})**

Because this process is **isentropic** (reversible adiabatic), no heat is transferred ($Q = 0$). According to the Non-Flow Energy Equation (NFEE), the work done is equal to the negative change in internal energy:

$$\frac{W_{1-2}}{m} = -\Delta U = -C_v (T_2 - T_1) = \left(0.7186 \frac{\text{kJ}}{\text{kg-K}} \right) (543.4342 - 300) \text{ K} = -174.9318 \frac{\text{kJ}}{\text{kg}}$$

The negative sign tells us that work was **inputted into** or required by the system to compress the air.

- **Process 2-3: Isobaric Heat Addition (W_{2-3})**

$$\frac{W_{2-3}}{m} = R(T_3 - T_2) = \left(0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(2400 - 543.4342)\text{K} = 532.8344 \frac{\text{kJ}}{\text{kg}}$$

*The positive sign tells us that the gas did work **on the surroundings** as it expanded due to the heat addition.*

- **Process 3-1: Isometric Heat Rejection (W_{3-1})**

This process occurs at a **constant volume** ($V = C$), meaning the air is trapped in a rigid space while cooling down.

$$\frac{W_{3-1}}{m} = 0 \frac{\text{kJ}}{\text{kg}}$$

By summing the total work transfers along the process paths $\left(\Sigma W_{NET} = \frac{W_{1-2}}{m} + \frac{W_{2-3}}{m} + \frac{W_{3-1}}{m}\right)$, the net specific work output equals:

$$\Sigma W_{NET} = -174.9318 \frac{\text{kJ}}{\text{kg}} + 532.8344 \frac{\text{kJ}}{\text{kg}} + 0 \frac{\text{kJ}}{\text{kg}} = 357.9026 \frac{\text{kJ}}{\text{kg}}$$

Thermal Efficiency (e_{cycle})

Using these values, the cycle thermal efficiency is determined by:

$$e_{cycle} = \frac{\Sigma W_{NET}}{\Sigma Q_A} = \frac{357.9026 \frac{\text{kJ}}{\text{kg}}}{1868.0765 \frac{\text{kJ}}{\text{kg}}} = 19.16\%$$

This sample problem provides important insights into real-world engine design. The power cycle analyzed here achieves a relatively low thermal efficiency of only 19.16%, despite reaching a peak combustion temperature of 2400K and a peak internal pressure of 1200kPa.

In industrial practice, sustaining an operational thermal load of 2400 K requires specialized, high-cost alloys to ensure the structural integrity of the cylinder walls. Generating less than 20% efficiency from such a high thermal load is economically impractical compared to a standard four-stroke cycle, which uses an additional isentropic expansion step to extract more mechanical work from the gas before rejecting heat.