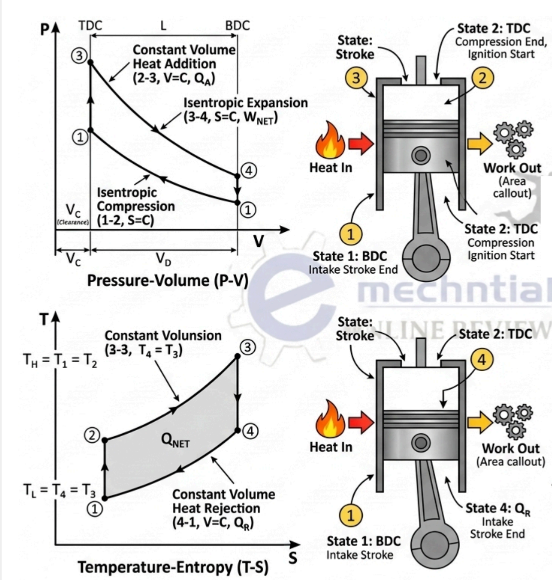
Thermodynamic Cycle

In thermal engineering, a **thermodynamic cycle** is defined as a closed series of distinct thermodynamic processes through which a working fluid continuously transmits heat and work while varying its properties (such as pressure, volume, temperature, and entropy), ultimately returning the system to its precise original state. Because the initial and final states of a complete cycle are identical, the net change in any state function or intrinsic property (such as internal energy, ΔU_{net} = 0, or enthalpy, ΔH_{net} = 0) over the entire cycle is zero.

A classic practical example of a thermodynamic cycle is the **Internal Combustion Engine (Otto Cycle)**, which consists of four sequential steps:

1. **Isentropic Compression:** The air-fuel mixture is compressed reversibly and adiabatically within the cylinder, increasing its pressure and temperature.
2. **Isochoric Heat Addition:** The fuel ignites rapidly at constant volume, causing a sharp spike in pressure and temperature.
3. **Isentropic Expansion (Power Stroke):** The high-pressure gas expands reversibly and adiabatically, pushing the piston downward and producing useful mechanical work.
4. **Isochoric Heat Rejection:** The exhaust valve opens, releasing residual heat to the atmosphere at a constant volume, returning the fluid properties to the intake state.



Engineers visualize these cycles on a **Pressure-Volume (P-V) diagram** to track how thermal energy released from fuel is mechanically converted into shaft work. The enclosed area traced by the process curves on a PV diagram represents the network output of the cycle.

SECOND LAW OF THERMODYNAMICS

While the First Law of Thermodynamics dictates the conservation of energy, the **Second Law of Thermodynamics** establishes limits on the direction of heat transfer and the efficiency of energy conversion. It states that not all thermal energy supplied to a thermodynamic system can be converted into an equivalent amount of useful work—some portion of that energy must always be rejected as low-grade waste heat to the surroundings. Consequently, this law fundamentally demonstrates the absolute physical impossibility of creating a "perpetual motion machine of the second kind" (a machine that runs indefinitely by converting all absorbed heat entirely into work).

Kelvin-Planck Statement:

Named after the Scottish mathematician and physicist Lord Kelvin (William Thomson) and the German physicist Max Planck, this statement centers on the concept of thermal efficiency in power-producing devices. It states that **it is impossible for any device operating on a thermodynamic cycle to receive heat from a single thermal reservoir and produce a net amount of work.** In simpler terms, a heat engine cannot be 100% efficient.

Consider a Steam Power Plant operating in a Simple Rankine Cycle, the cycle shown below violates Kelvin-Planck statement.

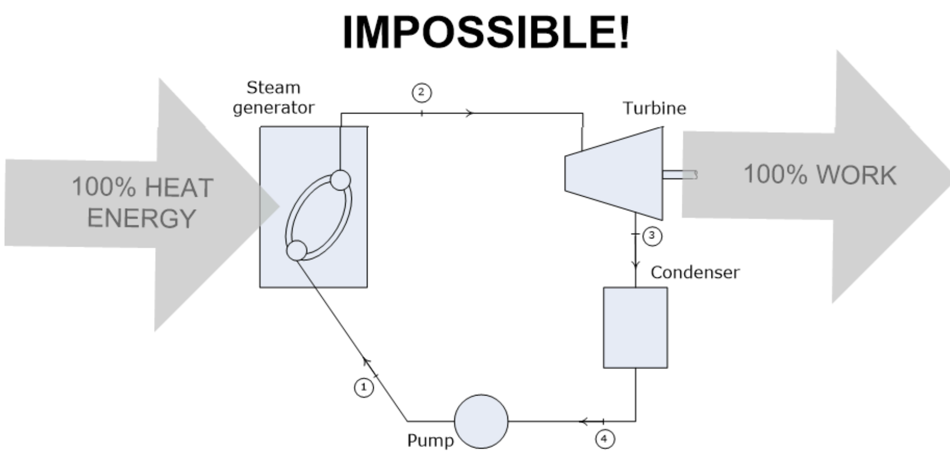


FIGURE 1: A steam cycle contradicting the Kelvin-Planck Statement

Consider a steam power plant operating on a Simple Rankine Cycle. If a design attempts to completely eliminate the condenser—meaning the steam passes through the boiler and turbine, and all heat input (Q_A) is converted entirely into shaft work (W_{net}) without rejecting any heat (Q_R = 0) to a cooling medium—the cycle directly **violates the Kelvin-Planck statement** and cannot exist in reality.

Clausius Statement:

Coined by Rudolf Clausius, this statement is based on the natural thermodynamic tendency of heat to spontaneously flow only from a high-temperature region to a low-temperature region. It states that **it is impossible to construct a self-acting device operating in a cycle that transfers heat from a colder body to a hotter body without the aid of external work.**

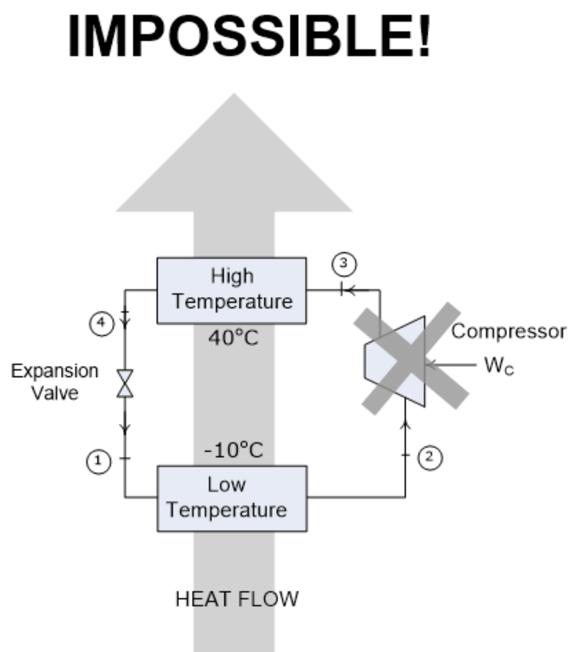


FIGURE 2: A refrigeration cycle contradicting the Clausius Statement

Consider a Simple Refrigeration Cycle, the cycle shown below violates Clausius Statement. Consider a Simple Refrigeration Cycle. Heat cannot spontaneously jump from the cold interior of a refrigerator to the warm room outside. If a diagram shows a refrigerator transferring heat (Q_R) from a cold body to a hot body while requiring zero mechanical power input from a compressor (W = 0), this cycle directly **violates the Clausius statement**. External work input is mandatory to reverse the natural direction of heat flow.

CYCLE ANALYSIS:

Equations:

(a) PVT Relationships

Applicable for any ideal gas processes

$$PV = mRT$$

Applicable for P=C, V=C and T=C only

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = mR = constant$$

For isentropic process $PV^k=C$, if special polytropic process $PV^n=C$, change k to n

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

(b) Heat Added, Q_A

$$Q_A = \Sigma + Q_A \mid_{cycle}$$

(c) Heat Rejected, Q_R

$$Q_R = \Sigma - Q_R \mid_{cycle}$$

(d) Net Work, W_{net}

$$W_{net} = \left|Q_A\right| - \left|Q_R\right| = \Sigma W_{NF} = \oint P\, dV$$

(e) Cycle Thermal Efficiency, e_{cycle}

$$e_{cycle} = \frac{W_{net}}{Q_A} x\ 100\% \ = \frac{|Q_A| - |Q_R|}{Q_A} x\ 100\%$$

$$e_{cycle} = 1 - \frac{Q_R}{Q_A} x\ 100\%$$

(f) Cycle Mean Effective Pressure, P_m

Cycle Mean Effective Pressure is the pressure developed by the cycle and is equivalent to work net divided by the volume displacement.

$$P_m = \frac{W_{net}}{V_D}$$

Where: V_D = volume displacement

$$P_m = \frac{W_{net}}{\left(V_{max} - V_{min}\right)}$$

The higher the value of Cycle Mean Effective Pressure means higher work produced per cycle and higher force per cycle output.

CYCLE ANALYSIS:
The following are the ratios that are used to describe a specific thermodynamic process in a cycle.

Expansion Ratio:	$r_{e_{process}} = \frac{V_{max}}{V_{min}}$	Measures the volumetric expansion during a specific power-producing process.
Compression Ratio:	$r_{k_{process}} = \frac{V_{max}}{V_{min}}$	Compares the maximum volume of the cylinder (BDC) to the minimum volume (TDC) during the compression stroke.
Cut-off Ratio:	$r_{c_{process}} = \frac{V_{max}}{V_{min}}$	Tracks the change in volume during a constant-pressure heat addition process (prominent in Diesel cycles).
Pressure Ratio:	$r_{p_{process}} = \frac{P_{max}}{P_{min}}$	Defines the mechanical pressure multiplication across a process (such as a compressor stage in a Brayton gas turbine cycle).

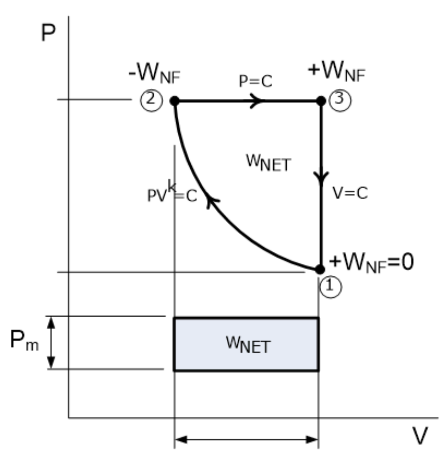
Sample Problem
Consider a three-process air-standard power cycle given by:

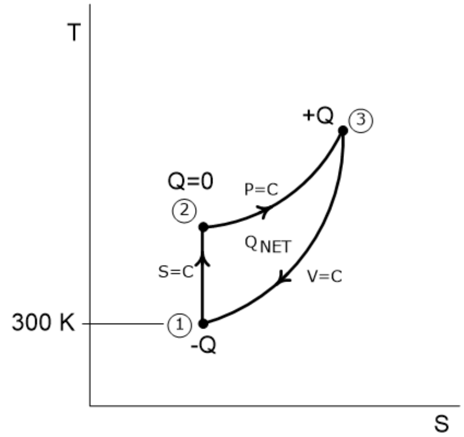
Process 1-2: Isentropic compression
Process 2-3 Constant pressure heat addition
Process 3-1: Constant volume heat rejection

The initial pressure and temperature of the air is P_1 = 150 kPa and T_1=300 K, respectively and the pressure at the end of the isentropic compression P_2=1200 kPa, Determine:

(a) The work and heat transfer for each process in kJ/kg
(b) The isentropic compression ratio, r_k
(c) The thermal efficiency of the cycle
(d) The mean effective pressure, in kPa
(e) Show the cycle on Ts and Pv coordinate

Solution:





Solving for the work in each process:

Process 1- 2: Isentropic Process

$$W_{NF_{1-2}} =- \Delta U =- mC_v(T_2 - T_1)$$

Process 1-2: $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{V_1}{V_2}\right)^{k-1}$

$$\frac{T_2}{300\ K} = \left(\frac{1200\ kPa}{150\ kPa}\right)^{\frac{1.4-1}{1.4}}$$

$$T_2 = 543.4342\ K$$

$$\frac{W_{NF_{1-2}}}{m} =- \left(0.7186\ \frac{kJ}{kg-K}\right)(543.4342\ -\ 300)K$$

$$\frac{W_{NF_{1-2}}}{m} =- 174.9318\ \frac{kJ}{kg}$$

Process 2-3: Isobaric Process

$$W_{NF_{2-3}} = mR(T_3 - T_2)$$

Process 1-2: $\frac{P_1}{T_1} = \frac{P_3}{T_3}$

$$\frac{150\ kPa}{300\ K} = \frac{1200\ kPa}{T_3}$$

$$T_3 = 2400\ K$$

$$\frac{W_{NF_{2-3}}}{m} = \left(0.287\ \frac{kJ}{kg-K}\right)(2400\ -\ 543.4342)K$$

$$\frac{W_{NF_{2-3}}}{m} = 532.8344\ \frac{kJ}{kg}$$

Process 3-1: Isometric Process

$$\frac{W_{NF_{3-1}}}{m} = 0\frac{kJ}{kg}$$

Solving for the heat in each process:

Process 1- 2: Isentropic Process

$$\frac{Q_{1-2}}{m} = 0$$

Process 2-3: Isobaric Process

$$Q_{2-3} = mC_p\left(T_3 - T_2\right)$$

$$\frac{Q_{2-3}}{m} = \left(1.0062\ \frac{kJ}{kg-K}\right)(2400\ -\ 543.4342)K$$

$$\frac{Q_{2-3}}{m} = 1868.0765\frac{kJ}{kg}$$

Process 3-1: Isometric Process

$$Q_{3-1} = mC_v\left(T_1 - T_3\right)$$

$$\frac{Q_{3-1}}{m} = \left(0.7186\ \frac{kJ}{kg-K}\right)(300\ -\ 2400)K$$

$$\frac{Q_{3-1}}{m} =-\ 1509.06\frac{kJ}{kg}$$

Solving for the isentropic compression ratio

$$r_{k_{1-2}} = \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}} = \left(\frac{1200\ kPa}{150\ kPa}\right)^{\frac{1}{1.4}}$$

$$r_{k_{1-2}} = 4.4164$$

Solving for the thermal efficiency

$$e_{cycle} = \frac{\Sigma W_{net}}{Q_A}x\ 100\% = \frac{(-174.9318+532.8344+0)\frac{kJ}{kg}}{1868.0765\frac{kJ}{kg}}x\ 100\%$$

$$e_{cycle} = 19.16\%$$

The cycle exhibits a relatively low thermal efficiency (19.16%) despite reaching a peak combustion temperature of 2400 K and a high peak pressure of 1200 kPa. In practical engine design, sustaining a 2400 K thermal load requires expensive, high-grade alloys. Generating only 19.16% efficiency from such a high thermal load is economically impractical compared to a standard Otto or Diesel cycle.

Solving for the Cycle Mean Effective Pressure,

$$P_m = \frac{e_{cycle} Q_A}{\left(V_1 - V_2\right)}$$

$$P_1 V_1 = mRT_1$$

$$(150\ kPa)\frac{V_1}{m} = \left(0.287\ \frac{kJ}{kg-K}\right)(300\ K)$$

$$\frac{V_1}{m} = 0.5740\ \frac{m^3}{kg}$$

$$r_{k_{1-2}} = \frac{V_1}{V_2} = \frac{0.5740\ \frac{m^3}{kg}}{V_2} = 4.4164$$

$$V_2 = 0.1299\ \frac{m^3}{kg}$$

$$P_m = \frac{(0.1916)\left(1868.0765\ \frac{kJ}{kg}\right)}{(0.5740-0.1299)\frac{m^3}{kg}}$$

$$P_m = 805.95\ kPa$$

The cycle produces a relatively high Mean Effective Pressure (806.09 kPa). This tells an engineer that despite its poor fuel economy, the engine can produce a high force output per unit of displacement volume. This parameter helps engineers balance efficiency against the structural size and weight constraints required for specific applications.

The Third Law of Thermodynamics

Developed by German chemist Walther Nernst (1864–1941) around the year 1905, the **Third Law of Thermodynamics** formalizes the baseline boundaries of temperature and entropy. It states:

> *"It is impossible by any procedure, no matter how idealized, to reduce the temperature of any system to the absolute zero in a finite number of operations."*

This law establishes that while engineers can approach absolute zero (0 K) through advanced cryogenic multi-stage systems, actually reaching this point would require an infinite number of thermodynamic steps or operations.

Carnot Model

The Carnot cycle serves as the theoretical bridge that allows us to mathematically define and understand the physical limits of the **Third Law of Thermodynamics**.

Carnot cycle efficiency: $$e = 1 - \frac{T_L}{T_H}$$

The mathematical formulation for Carnot efficiency reveals that an efficiency of unity (e = 1) can only occur when the temperature of the low-temperature heat sink (T_L) drops precisely to zero ($T_L = 0$). This value represents the lowest temperature conceivable, known as **absolute zero**, because an engine efficiency greater than unity (e > 1) is physically unthinkable. This establishes absolute zero as the lowest conceivable temperature boundary in the universe. It provides the inductive reference point upon which our absolute thermodynamic temperature scale is built.

No real-world or theoretical engine operating between two specific thermal reservoirs can have an efficiency greater than the corresponding Carnot engine operating between those same temperature limits. The Carnot efficiency establishes the absolute upper limit for power-cycle performance. Any claim of a cycle exceeding this value indicates a direct violation of thermodynamic principles.