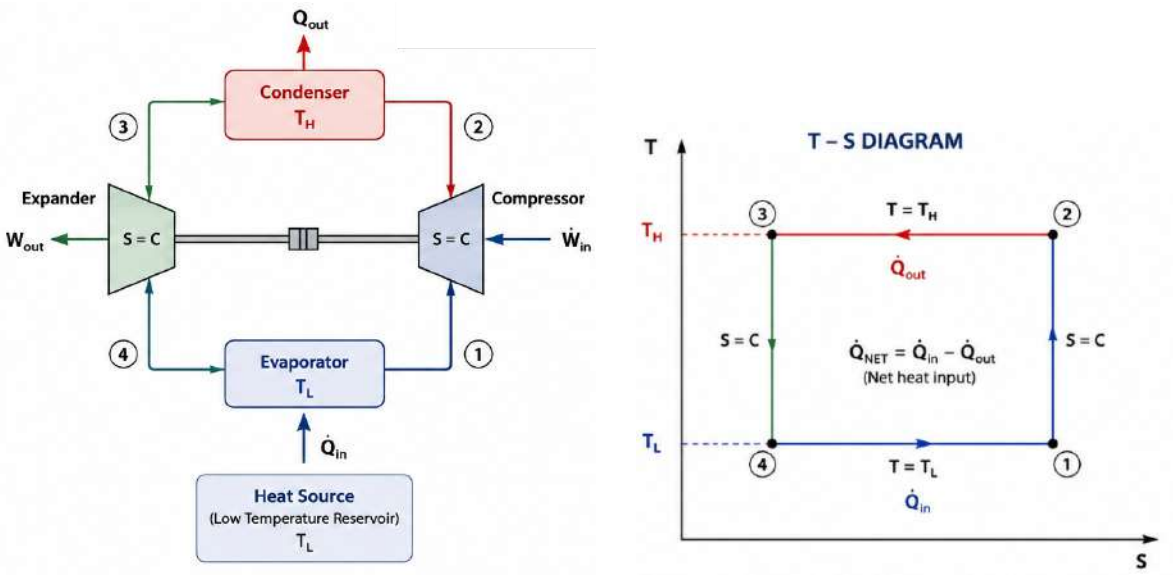
The Reversed Carnot Cycle

The reversed Carnot cycle serves as the theoretical benchmark for mechanical-compression refrigeration systems. It represents an ideal, albeit physically impossible, cycle with the highest theoretically possible thermal efficiency for any engine or device operating between two specific temperatures. For refrigeration, which can be thought of as a **heat engine working in reverse**, the cycle's primary function is to extract heat from a cold body and expel it to a hot body.



The figure illustrates the schematic and Temperature-Entropy (T-s) diagram of this cycle. By analyzing this cycle, engineers establish a standard of comparison for real-world **ideal vapor-compression cycles**, setting realistic goals for system improvements.

Coefficient of Performance (COP) as an Efficiency Parameter

Unlike standard heat engines, whose efficiency is defined as work output divided by heat input, a refrigeration system's effectiveness is measured by the ratio of the desired cooling effect (output) to the energy input required (input). This parameter is known as the **Coefficient of Performance, or COP**. In its most basic form, the desired output is the **rate of heat removal**, and the energy input is the power supplied to the **compressor**. A higher COP indicates a more efficient system, using less energy to achieve a given cooling capacity.

The process begins with heat absorption, Q_{in}, as represented by the area under the low-temperature line (state 2 to 1 on T-s diagram):

$$Q_{in} = \int T \cdot ds = T_L\left(s_2 - s_1\right) = T_L \Delta s$$

Heat rejection, Q_{out}, occurs as represented by the area under the high-temperature line (state 3 to 4):

$$Q_{out} = \int T \cdot ds = T_H\left(s_3 - s_4\right) = T_H \Delta s$$

Since the compressor and expander work for the cycle can be linked by the areas, the net work input, W_{NET}, is defined as:

$$W_{NET} = Q_{out} - Q_{in}$$

The COP equation is formulated as:

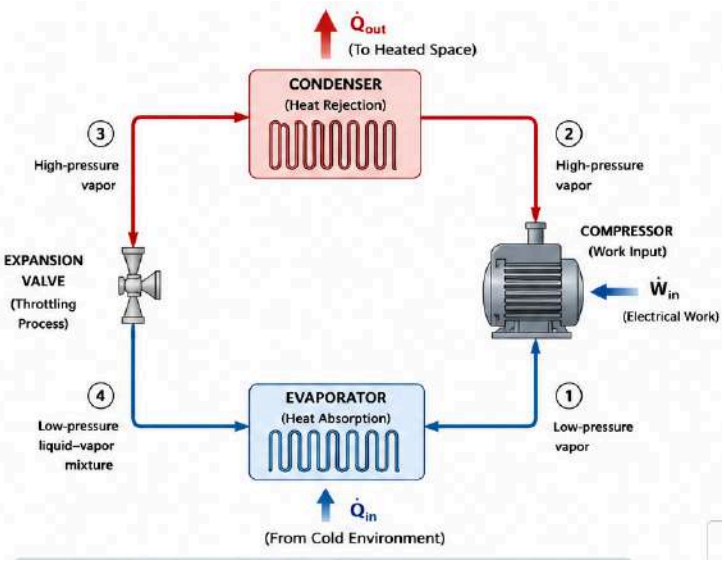
$$COP_{refrigeration} = \frac{Output}{Input} = \frac{Q_{in}}{W_{NET}} = \frac{Q_{in}}{Q_{out} - Q_{in}}$$

By substituting the T-Δs expressions into the equation and canceling the entropy terms, we derive the definitive temperature-based relationship for an ideal reversed Carnot cycle:

$$COP_{refrigeration} = \frac{T_L}{T_H - T_L}$$

Performance of a Heat Pump

A heat pump is a specific application of the reversed Carnot cycle, distinguished by its operational objective. While a refrigeration system is designed to provide cooling by extracting heat from a space, a heat pump is utilized to provide **heat to a space** by rejecting heat into it. Therefore, for a heat pump, the surroundings at low temperature act as the heat source, and the high-temperature space is where the desired effect (heating) occurs.



Performance Factor, PF (COP of Heating)

The effectiveness of a heat pump is often quantified by a similar ratio of desired output to energy input, sometimes referred to as the **Performance Factor, PF**. This parameter is analogous to the COP of refrigeration, but its desired "output" is heat rejection at high temperature, Q_{OUT}, rather than heat absorption. The net work input remains the same, W_{NET}. This leads to the fundamental equation:

$$\text{PF} = \frac{Output}{Input} = \frac{Q_{out}}{W_{NET}} = \frac{T_H \Delta s}{T_H \Delta s - T_L \Delta s}$$

Referencing the Carnot T-s expressions again:

$$\text{PF} = \frac{T_H}{T_H - T_L}$$

A crucial mathematical relationship links the performance factor of a heat pump to the COP of a refrigeration cycle when both are operating between the same two temperatures:

$$\frac{T_H}{T_H - T_L} = \frac{T_L + (T_H - T_L)}{T_H - T_L} = \frac{T_L}{T_H - T_L} + \frac{T_H - T_L}{T_H - T_L}$$

$$\text{PF} = COP_{refrigeration} + 1$$

It means that, theoretically, an ideal heat pump is always more than 100% efficient based on standard COP metrics, as it is designed to reject a quantity of heat that is inherently equal to the sum of the heat absorbed plus the compressor work input.

Tons of Refrigeration

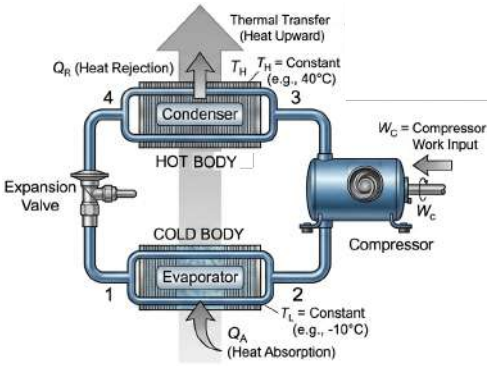
In practical engineering applications, cooling capacity is often not stated directly in kilowatts (kW) but using a historical unit known as **Tons of Refrigeration**. This unit is defined as the total amount of heat required to melt exactly one ton of ice within a single day (24 hours). To facilitate standard engineering calculations, critical conversion factors are established as:

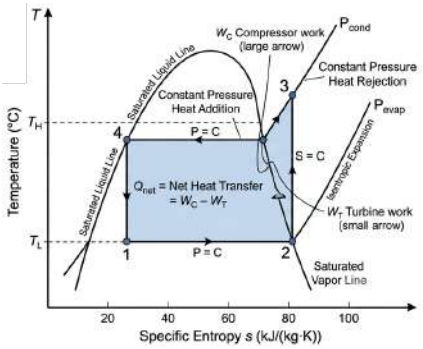
1 ton = 3.516 kW = 200 Btu/min

Energy Efficiency Rating, EER

While COP is a common metric in thermodynamic analysis, other regional standards exist for labeling commercial equipment. One such standard is the **Energy Efficiency Rating, EER**. The EER is defined as the specific number of BTUs of heat that can be removed from a load utilizing precisely 1 Watt-hour of energy. Similar to COP, a higher EER signifies a more efficient unit. A linear relationship exists between EER and COP, which allows for conversion during analysis:

EER = 3.41COP





A standard cycle on this diagram is formed by four key processes:

- **Process 1-2 (Evaporation):** Occurs at constant low pressure in the evaporator as represented by a horizontal line (P=C) from state 1 to 2.
- **Process 2-3 (Compression):** An isentropic mechanical-compression process in the compressor. It follows a path parallel to lines of constant entropy (S=C), typically moving upwards and to the right into the superheated vapor region (refer to image_63.png's diagram 1 for state definitions).
- **Process 3-4 (Condensation):** A heat rejection process that occurs at constant high pressure in the condenser. This is represented by a horizontal line (P=C) from state 3 to 4.
- **Process 4-1 (Expansion):** An **isenthalpic process** that takes place across the expansion device, such as a thermostatic valve. As there is no heat exchange or work, enthalpy remains constant, resulting in a vertical line down from state 4 to 1 (h=C).

Capacity Calculations

- **Rate of mass flow of refrigerant:** This is often the first value needed and is generally calculated from known refrigeration capacity.
- **Work of Compressor (W_C):** The rate of work input to the compressor is proportional to the refrigerant mass flow rate, mR, and the change in enthalpy across the compressor. Enthalpy at state 3 is superheated vapor and state 2 is saturated or superheated vapor:

$$W_C = m_R(h_3 - h_2)$$

- **The Refrigerating or Cooling Capacity in the Evaporator (Q_A):** This represents the rate of desired cooling and is based on the enthalpy change in the evaporator. Enthalpy at state 2 is saturated vapor and state 1 is a liquid-vapor mixture:

$$Q_A = m_R(h_2 - h_1)$$

- **The Heat Rejected at the Condenser (Q_R):** This is the rate of heat rejection to the high-temperature environment. The provided formula $Q_R = -m_R(h_3 - h_4)$ likely uses a sign convention where heat rejected is treated as negative. To define the rejected heat quantity, we can also use:

$$Q_R = m_R(h_3 - h_4)$$

- **Isenthalpic Processes at the Expansion Valve:** As discussed, ideal behavior in the expansion valve means enthalpy at the inlet (state 4, saturated liquid) is equal to enthalpy at the outlet (state 1, liquid-vapor mixture):

$$h_4 = h_1$$

The Pressure-Enthalpy (P-h) Diagram

The Pressure-enthalpy diagram, frequently referred to as the P-h diagram, is a powerful graphical tool widely used in the refrigeration industry to visualize a cycle's properties and processes. Unlike the T-s diagram which is useful for entropy analysis, the P-h diagram provides direct readouts for pressure, enthalpy, temperature, specific volume, and specific entropy.

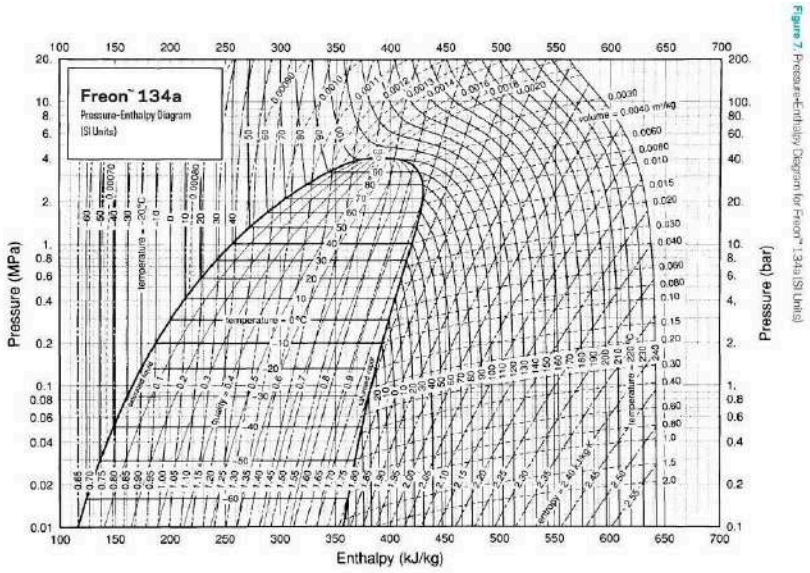


Figure 7. Pressure-Enthalpy Diagram for Freon™ 134a (SI Units)

A typical P-h plot includes constant entropy (S=C) lines that indicate specific entropy, constant specific volume (v=C) lines, and constant temperature (T=C) lines

Using the P-h diagram and specific enthalpy values, we can calculate the energetic performance of actual cycles.

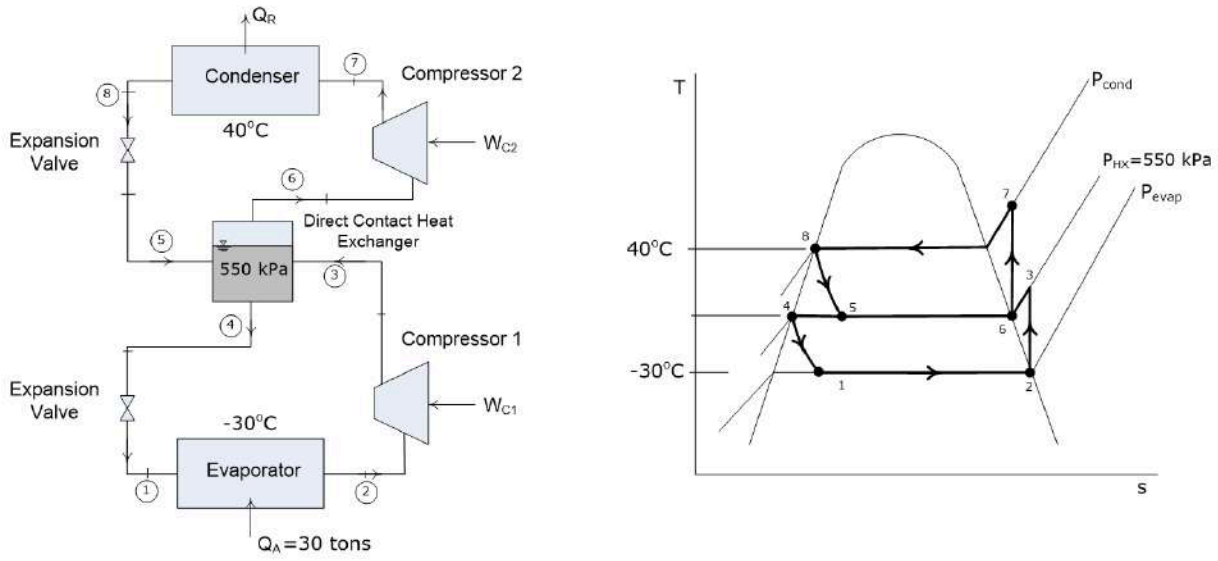
Multi-Pressure Vapor Compression Cycles

A standard single-stage vapor compression refrigeration cycle operates between two discrete pressures: a low evaporator pressure and a high condenser pressure. However, many industrial and commercial engineering applications require simultaneous cooling at different temperatures, or demand high-temperature lifts that make single-stage systems economically or thermodynamically unfeasible. To address these demands, engineers implement **multi-pressure vapor compression cycles**.

By technical definition, a multi-pressure system is a refrigeration setup featuring two or more low-side pressures. In this context, "low-side pressures" refer explicitly to the operating pressures of the refrigerant streams situated anywhere between the initial expansion valve outlet and the intake manifold of the high-stage compressor. These modern configurations utilize modifications and refinements to improve thermodynamic performance over basic vapor compression arrangements. Two primary configurations dominate this landscape: **direct contact cascade refrigeration systems** and **multi-evaporator refrigeration systems**.

Direct Contact Cascade Refrigeration Systems

A direct contact cascade refrigeration system employs staging to handle large temperature intervals while minimizing compressor work. This system divides the total lift into a low-stage cycle and a high-stage cycle, which interface directly via an open-type chamber called a **direct contact heat exchanger** (or flash intercooler).



As mapped on the mechanical layout and Temperature-Entropy (T-s) plot in the attached reference, the operational states track as follows:

- **Low-Stage Loop:** Saturated liquid refrigerant expands through the low-stage expansion valve down to the low evaporator pressure (P_{evap}) corresponding to a low temperature (e.g., -30°C). The refrigerant flows into the low-stage evaporator, absorbing the primary refrigeration load (Q_{A1} = 30 tons). The leaving vapor (State 2) enters Compressor 1, where it undergoes isentropic compression to the intermediate inter-stage pressure (P_{HX} = 550 kPa) before discharging directly into the mixing chamber at State 3.
- **High-Stage Loop:** High-pressure liquid exiting the condenser (40°C saturation temperature) is throttling-expanded through the high-stage expansion valve down to the intermediate pressure (State 5) and enters the direct contact heat exchanger. The warmer expanded fluid mixes intimately with the superheated vapor from Compressor 1 (State 3). This process desuperheats the low-stage discharge gas while simultaneously

flashing a portion of the liquid refrigerant. Saturated vapor is drawn out from the top of the heat exchanger (State 6) and enters Compressor 2, where it is compressed to the full condenser pressure (P_{cond}). The hot discharge vapor condenses back to liquid in the condenser, rejecting heat (Q_R) to the high-temperature sink. Liquid from the bottom of the intercooler (State 4) supplies the low-stage expansion loop.

Energy and Mass Balance Analysis

To evaluate the fluid ratios between the independent loops, a steady-flow thermal energy balance is applied directly across the boundary of the direct contact heat exchanger. Let m_3 and m_4 represent the mass flow rates circulating through the low-stage loop, and let m_5 and m_6 represent the mass flow rates circulating through the high-stage loop.

Under steady-flow criteria where mass and energy are conserved (E_{IN} = E_{OUT}), the governing equation is formulated as:

$$m_5 h_5 + m_3 h_3 = m_6 h_6 + m_4 h_4$$

Due to state identity configurations inside the saturated intercooler chamber, specific enthalpy identities are fixed based on phase:

- Upper Expansion Valve: $h_8 = h_5$

  Lower Expansion Valve: $h_4 = h_1$

- Saturated vapor states: $m_2 = m_4 = m_3$; $m_7 = m_5 = m_6$

Isolating the relative mass ratios yields the direct mass scaling factor between the high-stage and low-stage streams:

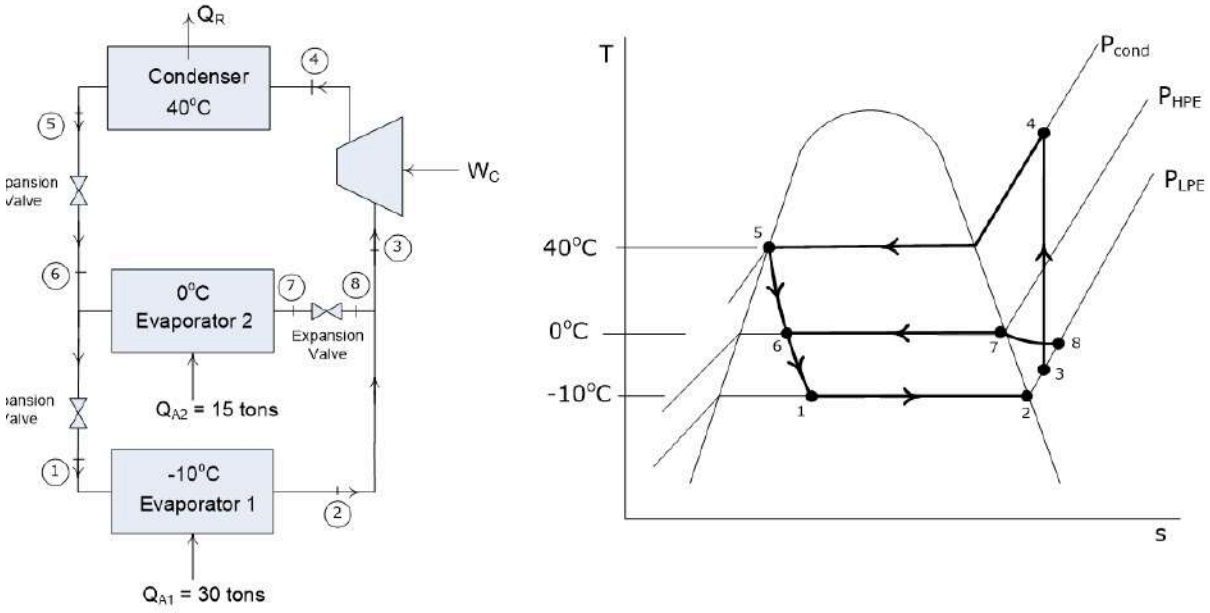
$$\frac{m_3}{m_7} = \frac{h_6 - h_5}{h_3 - h_4}$$

To minimize the combined total mechanical work input required by both compressors (W_{C1} + W_{C2}), the selection of the intermediate operating pressure cannot be arbitrary. For ideal gas behavior or symmetrical compression stages, the optimum inter-stage pressure (P_{HX}) inside the direct contact heat exchanger is mathematically derived as the geometric mean of the absolute evaporator and condenser pressures:

$$P_{HX} = \sqrt{P_{cond} P_{evap}}$$

Multi-Evaporators with One Compressor System

When an engineering facility requires refrigeration at multiple distinct temperatures—such as a supermarket requiring one temperature tier for fresh produce dairy chilling (0°C) and another lower tier for frozen food storage (-10°C)—installing separate single-stage systems can be inefficient. A **multi-evaporator system running on a single compressor** solves this problem by utilizing specialized expansion configurations.



Referring to the multi-evaporator layout and T-s plot in the attached reference, the operational states track as follows:

- Saturated high-pressure liquid leaves the condenser (40°C) and splits into two parallel distribution lines.
- **High-Temperature Evaporator Loop (Evaporator 2):** Liquid refrigerant undergoes expansion through an initial valve down to the intermediate pressure (P_{HP2}) matching the 0°C temperature zone, where it absorbs a refrigeration load of Q_{A2} = 15 tons. The exiting vapor at State 8 is throttled down through an expansion valve to match the absolute lowest system pressure (P_{evap}).
- **Low-Temperature Evaporator Loop (Evaporator 1):** The second liquid line expands directly down through its expansion valve to the absolute lowest system pressure (P_{evap}) corresponding to the -10°C temperature zone, where it absorbs a refrigeration load of Q_{A1}= 30 tons. The vapor leaves at State 2.

Before entering the single compressor inlet, the vapor stream leaving the low-temperature evaporator (State 2) and the throttled vapor stream leaving the high-temperature evaporator (State 8) merge at a physical pipe junction. The resulting mixture forms State 3, which defines the precise thermodynamic inlet condition for the compressor (W_C).

To find the specific enthalpy at State 3 (h_3), engineers must apply a localized conservation of mass and energy balance around this mixing junction where states 2, 8, and 3 converge.

The balance equation is written as: ($E_{IN} = E_{OUT}$)

$$m_8 h_8 + m_2 h_2 = m_3 h_3$$

By substituting the mass conservation identity ($m_3 = m_8 + m_2$) into the energy equation, we obtain:

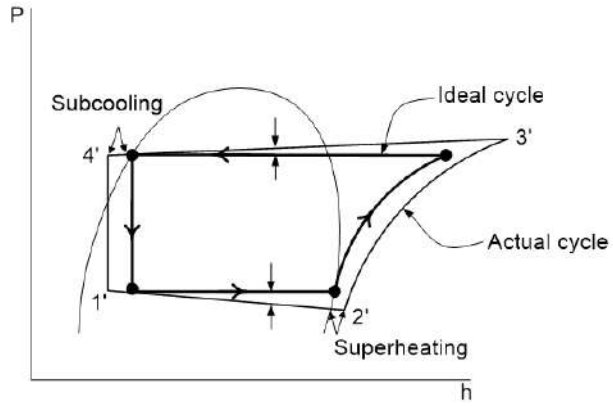
$$m_8 h_8 + m_2 h_2 = \left(m_8 + m_2\right) h_3$$

Solving directly for the mixed compressor inlet enthalpy (h_3) yields the weighted mass average of the merging enthalpies:

$$h_3 = \frac{m_8 h_8 + m_2 h_2}{m_2 + m_8}$$

The Actual Vapor-Compression Cycle

In practical refrigeration applications, systems deviate significantly from the idealized thermodynamic representations due to inherent real-world irreversibilities. Fluid friction accounts for considerable pressure drops as the working fluid flows through the evaporator tubes, condenser coils, interconnecting piping, and valving mechanisms. Additionally, stray heat transfers to or from the surrounding environment further warp the performance profile away from ideal behavior.



The Pressure-Enthalpy (P-h) graphic highlights these real-world distortions. Instead of following perfectly horizontal constant-pressure lines or idealized vertical isentropic paths, the actual cycle plots with distinct sloped pressure drops and non-isentropic compression lines.

Degree of Superheating (ΔT_{SH})

Superheating occurs when a vapor is heated to a temperature higher than its saturation temperature at a given pressure.

The degree of superheating is the difference between the **actual temperature of the superheated vapor leaving the evaporator** ($t_{2'}$) and the **saturated vapor temperature corresponding to the evaporator pressure** (t_{evap}).

$$\Delta T_{SH} = t_{2'} - t_{evap}$$

Superheating is vital in practice because it guarantees that no remaining liquid droplets enter the compressor suction manifold, which protects internal components from damaging liquid slugging events.

Degree of Subcooling (ΔT_{SC})

Subcooling is the cooling of a liquid refrigerant below its condensing temperature at a given pressure.

Subcooling represents the temperature difference between the **liquid saturation temperature corresponding to the condenser pressure** (t_{cond}) and the **actual temperature of the subcooled liquid prior to reaching the expansion valve** ($t_{4'}$).

$$\Delta T_{SC} = t_{cond} - t_{4'}$$

Subcooling increases the net refrigerating effect by minimizing flash gas formation during the throttling process, which maximizes the system's thermal capacity.

A refrigerant serves as the active working fluid within a closed-loop refrigeration cycle. It undergoes phase changes to alternate between absorbing heat at low pressures and rejecting heat at high pressures. For safe and reliable operation, a high-quality refrigerant should exhibit specific key characteristics:

- **Nontoxic:** Safe for operators and inhabitants in the event of an accidental leak.
- **Unreactive:** Chemically inert with system piping, compressor oil, and gaskets.
- **Stable:** Thermally resilient over the entire operating lifetime of the refrigeration machinery.
- **Ozone Safe:** Formulated with a low or zero Ozone Depletion Potential (ODP) to safeguard the environment.
- **Low Boiling Point:** Capable of evaporating at low temperatures under reasonable pressures.

Inorganic Compounds (The 700 Series)

Inorganic refrigerants are designated by a standard nomenclature system where the compound is prefixed with the letter **R-** followed by the number **7** combined with the substance's molecular weight.

H_2O	Water	R-718
CO_2	Carbon Dioxide	R-744
NH_3	Ammonia	R-717
SO_2	Sulfur Dioxide	R-764
Air	Atmospheric Air Mixture	R-729

Classification by Chemical Composition

Refrigerants are broadly subdivided based on their atomic arrangements:

- **(a) Halocarbons:** Formed from basic carbon skeletons bound to halogens (nonmetal elements such as chlorine, bromine, iodine, and astatine) along with variable trace impurities.
    - *Trichlorofluoromethane* (CC_3F): **R-11**
    - *Dichlorodifluoromethane* (CCl_2F_2): **R-12**
    - *Chlorodifluoromethane* ($CHClF_2$): **R-22**
    - *Tetrafluoroethane* ($C_2H_2F_4$): **R-134a**
- **(b) Hydrocarbons:** Pure organic compounds containing only carbon and hydrogen atoms.
    - *Methane* (CH_4)
    - *Ethane* (C_2H_6)
    - *Propane* (C_3H_8)

Product Thermal Load Calculations

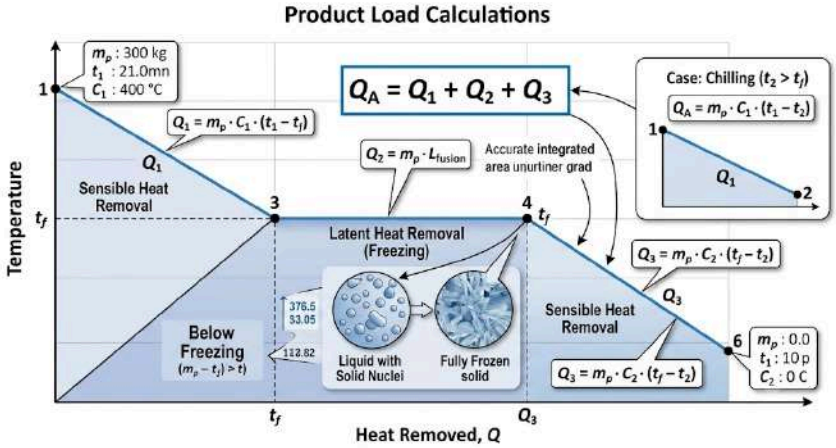
To properly size a refrigeration system's compressor and evaporator capacity (Q_A), engineers must compute the total thermal energy that must be removed from a product over a given timeframe. This energy budget depends heavily on whether the storage temperature drops below the product's unique freezing threshold.

Case A: Sensible Cooling Load Without Freezing

When a product is chilled but stays entirely above its freezing point ($t_2 > t_f$), the process involves purely sensible heat transfer. The load calculation uses the product's mass, specific heat capacity, and temperature differential:

$$Q_A = mC_p\Delta T$$

Case B: Multi-Stage Cooling Load With Freezing



When a product must be frozen and stored at temperatures far below its freezing point ($t_2 < t_f$), the total heat extraction (Q_A) spans three distinct operational stages:

1. **Sensible Cooling Above Freezing (Q_1):** Lowering the liquid or raw product temperature down to its freezing threshold (t_f) using the specific heat capacity above freezing (C_1).
2. **Latent Heat of Fusion Phase Change (Q_2):** Freezing the moisture content at a constant temperature (t_f) using the latent heat of fusion (L).
3. **Sensible Cooling Below Freezing (Q_3):** Sub-cooling the solid frozen mass down to the final storage room temperature (t_2) using the specific heat capacity below freezing (C_2).

Summing these individual loads ($Q_A = Q_1 + Q_2 + Q_3$) yields the comprehensive multi-stage load equation:

$$Q_A = m\left[C_1\left(t_1 - t_f\right) + L + C_2\left(t_f - t_2\right)\right]$$

Where:

- m = mass of the product (kg)
- C_1 = specific heat capacity above freezing (kJ/kg-°C)
- C_2 = specific heat capacity below freezing (kJ/kg-°C)
- L = latent heat of fusion (kJ/kg)
- t_1 = initial entering temperature of product (°C)
- t_f = freezing point temperature of product (°C)
- t_2 = final storage temperature of product (°C)