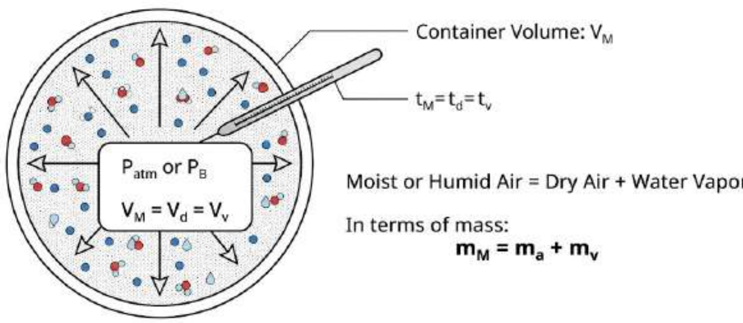
Psychrometrics

Psychrometrics is the branch of thermal science concerned with the measurement, evaluation, and thermodynamic analysis of water vapor content mixed within atmospheric air. In practical engineering contexts, ambient air is never completely pure or dry; it is modeled as a binary mixture composed of dry air (a static pseudo-pure gas mixture of nitrogen, oxygen, and trace gases) and an actively variable quantity of water vapor.



To establish a reference for mass and volume conservation within an isolated system, the total volume of the moist air mixture (V_m) is mathematically identical to the volume occupied by each constituent gas component independently ($V_a = V_v = V_m$). However, the total mass of the humid air mixture (m_H) is equal to the sum of the absolute mass of the dry air (m_a) and the mass of the suspended water vapor (m_v):

$$m_H = m_a + m_v$$

Dalton's Law of Partial Pressures

Based on Dalton's Law of partial pressures, the total barometric or atmospheric pressure (P_{atm} or P_B) exerted by a gas-vapor mixture is equal to the sum of the independent partial pressures exerted by each individual gas component as if it occupied the entire system volume alone at the mixture temperature. For a moist air system, this behavior is represented by the formula:

$$P_{atm} = P_B = P_a + P_v$$

Where P_a represents the partial pressure of the dry air component, and P_v represents the partial pressure of the water vapor component. Under typical atmospheric conditions, both components can be modeled as ideal gases, allowing their individual state properties to be calculated using the ideal gas equation of state:

$$P_a = \frac{m_a R_a T_a}{V_a} \qquad \text{and} \qquad P_v = \frac{m_v R_v T_v}{V_v}$$

The specific gas constants required for these ideal gas equations depend on the molecular weights of the respective components. For dry air, the specific gas constant (R_a) is 0.2870 kJ/kg-K. For water vapor, the specific gas constant (R_v) is 0.4615 kJ/kg-K.

Partial Vapor Pressure via Carrier's Equation

When direct psychrometric measurement of water vapor partial pressure (P_v) is not possible using basic sensors, it can be calculated using Dr. Carrier's empirical equations. These equations correlate the physical dry-bulb and wet-bulb temperatures to determine the vapor pressure relative to the saturation pressure at the wet-bulb temperature (P_{SW}).

- **SI Metric Units Specification:**

$$P_v = P_{SW} - \frac{(P_B - P_{SW})(t_d - t_w)}{1500 - 1.44\, t_w}$$ where pressures are in kPa and temperature in ºC

- **English Engineering Units Specification:**

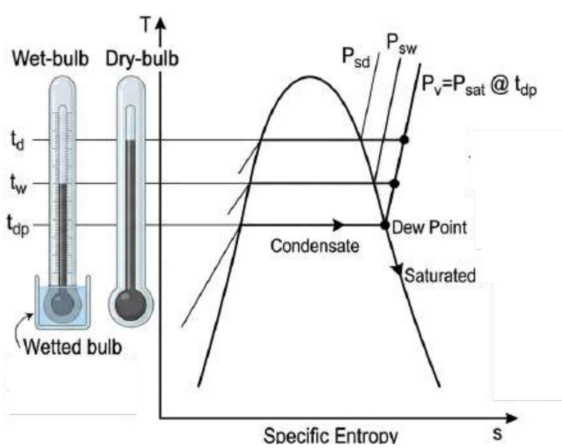
$$P_v = P_{SW} - \frac{(P_B - P_{SW})(t_d - t_w)}{2830 - 1.44\, t_w}$$ where pressures are in psia and temperature in ºF

Where:

- P_{SW} = Saturated vapor pressure evaluated specifically at the wet-bulb temperature (t_w).
- t_w = Wet-bulb temperature.
- t_d = Dry-bulb temperature.

Temperature Classifications in Psychrometrics

To accurately analyze thermodynamic processes on a Psychrometric Chart or a Temperature-Entropy (T-s) diagram, three distinct temperature scales must be clearly defined and understood.



Dry-Bulb Temperature (t_d)

The dry-bulb temperature is the actual ambient temperature of the moist air mixture measured using an ordinary, unmodified thermometer exposed directly to the gas-vapor mixture. It represents the true kinetic thermal energy state of the air and serves as the primary independent variable along the horizontal axis of standard psychrometric charts.

Wet-Bulb Temperature (t_W)

The wet-bulb temperature is measured using an ordinary thermometer whose sensing bulb is enclosed in a porous cloth wick or sleeve kept thoroughly moistened with liquid water. As unsaturated air flows over the wetted sleeve, water continuously evaporates from the wick into the passing air stream. Because evaporation requires latent heat, thermal energy is drawn from the thermometer bulb, causing its temperature to drop below the dry-bulb reading. The system eventually reaches a dynamic equilibrium where the convective sensible heat transferred into the bulb matches the latent heat removed by evaporation. This equilibrium reading is the wet-bulb temperature, and it serves as a measure of the air's evaporation potential.

Dew Point Temperature (t_{dp})

The dew point temperature is the thermal threshold where a moist air mixture becomes completely saturated with water vapor (100% relative humidity). If a gas-vapor mixture is cooled at a constant pressure and constant moisture mass fraction, its partial vapor pressure (P_v) eventually equals the saturation pressure of water (P_{sat}). The temperature at which this occurs is the dew point ($P_v = P_{sat}@t_{dp}$). Any further cooling below this value forces the excess water vapor to undergo a phase change, condensing out of the air stream as liquid droplets (dew or condensate).

Quantitative Moisture

Humidity Ratio or Specific Humidity (SH or ω)

The humidity ratio (or specific humidity) is an absolute moisture index defined as the ratio of the absolute mass of water vapor to the absolute mass of dry air present within a given sample volume. Unlike relative indices, this parameter remains constant during sensible heating or cooling processes unless water vapor is actively added to or condensed out of the system.

$$SH = \frac{m_v}{m_a}$$

The standard engineering units for this parameter are expressed as kilograms of water vapor per kilogram of dry air (kg_v/kg_a) or pounds of water vapor per pound of dry air (lb_v/lb_a).

This ratio can be rewritten using the partial pressures of the components:

$$SH = \frac{\frac{m_v R_v T_v}{V_v}}{\frac{m_a R_a T_a}{V_a}} = \frac{P_v}{P_a}\frac{R_a}{R_v}$$

Substituting the specific gas constants (R_a = 0.2870 kJ/kg-K and R_v = 0.4615 kJ/kg-K) yields the standard constant ratio (0.2870 / 0.4615 = 0.622). Combining this with Dalton's Law $\left(P_a = P_B - P_v\right)$gives the final equation used for board examinations:

$$SH = 0.622\left(\frac{P_v}{P_B - P_v}\right)$$

Relative Humidity (RH)

Relative humidity is a ratio that compares the actual moisture content of the air to the maximum moisture amount the air can hold at its current temperature. It is defined as the ratio of the partial pressure of water vapor (P_v) in the moist air sample to the saturation pressure of water vapor (P_{sd} or P_{sat}) evaluated at the identical dry-bulb temperature (t_d).

$$RH = \frac{P_v}{P_{sd}}$$

Because the saturation pressure of water vapor rises exponentially with temperature, heating an air sample without adding more moisture will cause its relative humidity to drop sharply. This occurs because the denominator (P_{sd}) increases while the numerator (P_v) stays constant.

Absolute Humidity (H_{abs})

Absolute humidity measures the actual mass of water vapor present within a unit volume of the moist air mixture. It represents the physical density of the water vapor component (ρ_v) within the gas-vapor space.

$$H_{abs} = \frac{m_v}{V_m} \qquad \text{expressed in } \frac{kg_v}{m_m^3}$$

Because dry air and water vapor occupy the exact same total mixture volume ($V_m = V_a = V_v$), absolute humidity can be linked to the dry air state properties using the ideal gas law:

$$V_a = \frac{m_a R_a T_a}{P_a} \qquad \Rightarrow \qquad H_{abs} = \frac{m_v}{\frac{m_a R_a T_a}{P_a}} = \frac{m_v}{m_v}\left[\frac{P_a}{R_a T_a}\right]$$

$$H_{abs} = SH\left[\frac{P_a}{R_a T_a}\right]$$

Saturation Ratio or Degree of Saturation (ϕ)

The saturation ratio (or degree of saturation) evaluates how close an air sample is to total saturation. It is defined as the ratio of the air sample's actual humidity ratio (SH) to its theoretical maximum humidity ratio if it were fully saturated (SH_S) at the same dry-bulb temperature.

$$\phi = \frac{SH}{SH_S} x\, 100\%$$

We can derive a direct relationship between the saturation ratio (ϕ) and relative humidity (RH) by expanding both specific humidity terms using their partial pressure components:

$$\phi = \frac{0.622\left(\frac{P_v}{P_B - P_v}\right)}{0.622\left(\frac{P_{sd}}{P_B - P_{sd}}\right)} = \frac{P_v}{P_{sd}}\left[\frac{P_B - P_{sd}}{P_B - P_v}\right]$$

$$\phi = RH\left[\frac{P_B - P_{sd}}{P_B - P_v}\right] x\ 100\%$$

Specific Volume of Dry Air (v_a)

In psychrometrics, specific volume (v_a) is conventionally expressed per unit mass of dry air rather than per unit mass of the total mixture. This convention simplifies calculations because the mass of dry air remains constant during heating, cooling, and humidification processes. Using the dry air ideal gas equation:

$$P_a V_a = m_a R_a T_d \qquad \Rightarrow \qquad v_a = \frac{V_a}{m_a} = \frac{R_a T_d}{P_a} \qquad \text{expressed in } \frac{m^3}{kg_a}$$

Substituting Dalton's Law ($P_a = P_B - P_v$), the equation becomes:

$$v_a = \frac{R_a T_d}{P_B - P_v}$$

Total Mixture Enthalpy (h)

The total enthalpy of a moist air mixture (H_M) is the sum of the enthalpy of the dry air carrier gas (H_a) and the enthalpy of the suspended water vapor (H_v):

$$H_M = H_a + H_v = m_a h_a + m_v h_v$$

Dividing by the constant mass of dry air (m_a) yields the specific enthalpy of the mixture (h) per kilogram of dry air:

$$h = \frac{H_M}{m_a} = h_a + \left(\frac{m_v}{m_a}\right) h_v$$

$$h = h_a + SH\left(h_v\right) \qquad \text{expressed in } \frac{kJ}{kg_a}$$

To calculate these values, a reference datum is established where the enthalpy of dry air is set to **0 kJ/kg** at **0°C**. This yields the following specific enthalpy equations:

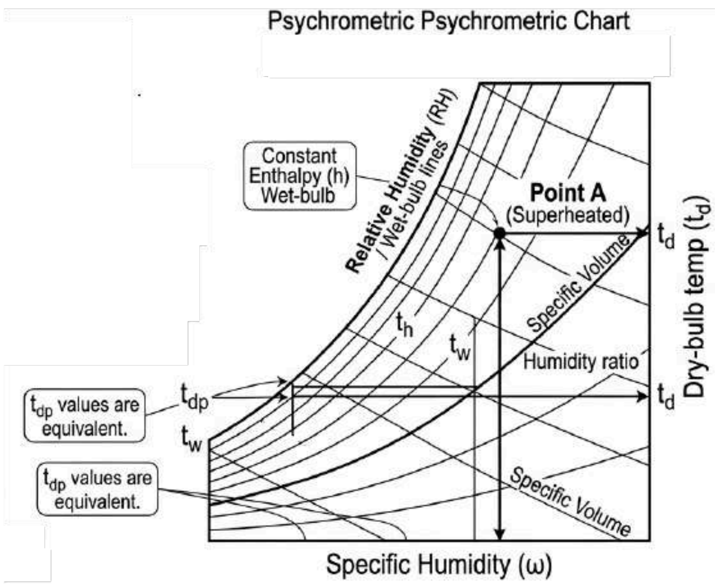
- **Dry Air Enthalpy Component:** $h_a = C_{pa}(t_d - 0) = C_{pa} t_d$
- **Water Vapor Enthalpy Component:** $h_v = h_g @ t_d$

If a steam table is not accessible during a board exam, the specific enthalpy of superheated water vapor can be approximated using this empirical formula:

$$h_v = 2501.3 + 1.82(t_d)$$

This linear approximation retains high accuracy for engineering applications, with a maximum error of less than **1.9 kJ/kg** across an operating range from **-20°C** to **100°C**.

The **Psychrometric Chart** provides a graphical layout of the thermodynamic properties of moist air at a constant total barometric pressure (typically calibrated to standard sea level, **101.325 kPa**).



A property state is defined by the intersection of these key gridlines:

- **Dry-Bulb Temperature (t_d):** Represented by straight, vertical lines across the chart.
- **Specific Humidity (SH):** Represented by horizontal lines that scale along the right-hand vertical axis.
- **Relative Humidity (RH):** Represented by curved lines that arch upward from the bottom left toward the top right. The outermost curve is the 100% relative humidity saturation curve.
- **Specific Enthalpy (h) & Wet-Bulb Temperature (t_w):** Mapped as diagonal lines sloping downward from the upper left toward the lower right.
- **Specific Volume (v_a):** Represented by steep, widely spaced diagonal lines.

Depression values measure how far an unsaturated air sample is from its saturation limits.

A. Wet-Bulb Depression

The wet-bulb depression is the temperature difference between the dry-bulb temperature and the wet-bulb temperature. It directly indicates air dryness; a larger gap means the air is drier and has a higher capacity for evaporative cooling.

Wet-Bulb Depression = $t_d - t_w$

B. Dew Point Depression

The dew point depression is the temperature difference between the dry-bulb temperature and the dew point temperature. It measures how much an air sample must be cooled at a constant pressure before condensation will begin to form.

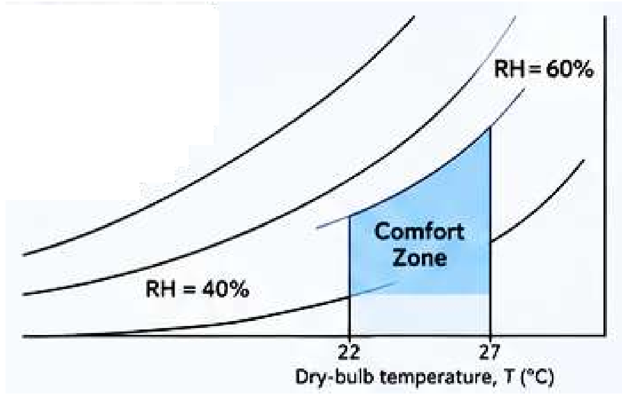
Dew Point Depression = $t_d - t_{dp}$

Air conditioning encompasses more than the simple reduction of sensible temperature. As a core branch of thermal engineering, air conditioning is defined as the simultaneous control of temperature, humidity, air velocity, and purity within a confined space to meet the requirements of human thermal comfort or industrial processing.

Human thermal comfort is governed by the body's ability to dissipate metabolic heat via radiation, convection, and latent evaporation. The human body continuously rejects heat, and its comfort is highly sensitive to the moisture content of the surrounding air. When humidity is excessively high, the evaporation of perspiration is severely retarded, leading to a sensation of oppressive warmth even at moderate temperatures. Conversely, overly dry environments accelerate evaporation, drying out mucous membranes and causing localized cooling discomfort.

The Comfort Zone and Psychrometric Mapping

The standard criteria for human thermal comfort are mapped onto a specialized thermodynamic chart known as the **Psychrometric Chart**. The chart evaluates parameters of moist air—which is treated as a binary mixture of ideal gases consisting of bone-dry air and superheated or saturated water vapor.

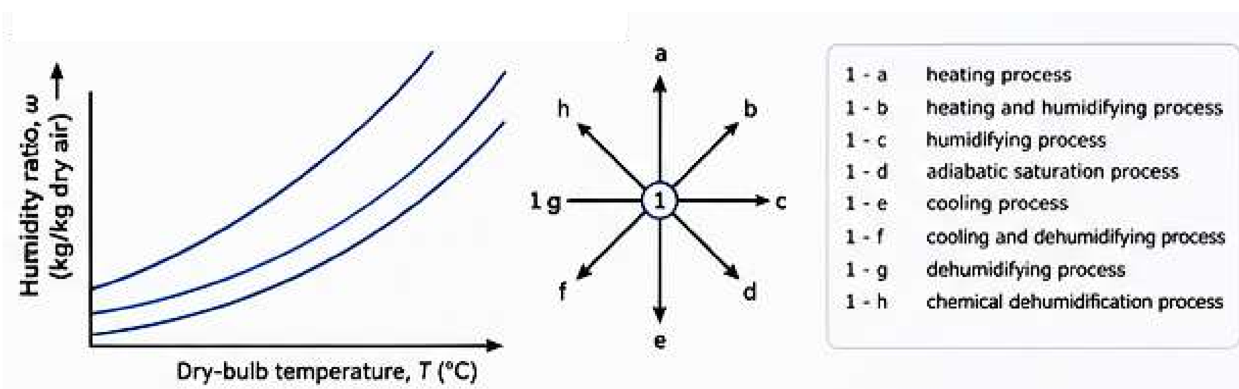


The thermodynamic **"Comfort Zone"** defines the specific boundary conditions where the majority of occupants experience thermal neutrality. Graphically, this envelope is bounded by:

- **Dry-Bulb Temperature (t_d):** Ranging strictly between **22ºCC and 27ºC**. This controls the sensible heat exchange between the human body and the air via convection and radiation.
- **Relative Humidity (RH):** Bounded between **40% and 60%**. This ensures an optimal vapor pressure differential between human skin and the atmosphere, allowing adequate but non-excessive latent heat dissipation through sweat evaporation.

Psychrometric Chart

Every thermodynamic alteration of a moist air mixture can be plotted as a directional state-change vector originating from an initial state point (State 1). Understanding the orientation of these vectors is vital for rapidly identifying processes during board examinations:

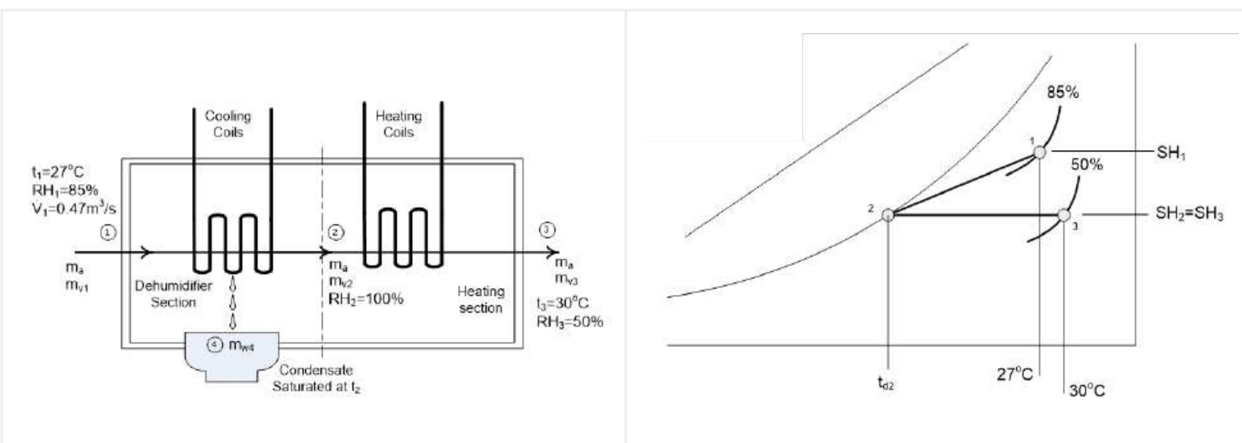


- **1-a: Pure Heating:** A horizontal line shifting directly to the right. The dry-bulb temperature increases while the humidity ratio (moisture content) remains perfectly constant.
- **1-b: Heating and Humidifying:** A diagonal vector pointing upward and to the right. Both the dry-bulb temperature and the absolute humidity ratio increase simultaneously (e.g., steam humidification in a heated air duct).

- **1-c: Pure Humidifying:** A vertical vector pointing straight up. The moisture content increases while the dry-bulb temperature remains unchanged (isothermal humidification).
- **1-d: Adiabatic Saturation (Evaporative Cooling):** A diagonal vector pointing upward and to the left, tracking parallel to lines of constant wet-bulb temperature or constant enthalpy. The dry-bulb temperature drops as sensible heat is converted into latent heat to vaporize water without any external heat transfer.
- **1-e: Pure Cooling:** A horizontal line shifting directly to the left. The dry-bulb temperature decreases while the absolute moisture mass ratio remains constant (sensible cooling above the dew point).
- **1-f: Cooling and Dehumidifying:** A diagonal vector pointing downward and to the left. As air is cooled below its initial dew point temperature, water vapor condenses out of the mixture, dropping both the dry-bulb temperature and the humidity ratio.
- **1-g: Pure Dehumidifying:** A vertical vector pointing straight down. Moisture is extracted from the air stream while keeping the dry-bulb temperature static.
- **1-h: Chemical Dehumidification:** A diagonal vector pointing downward and to the right. As air passes through a solid desiccant bed, moisture is chemically absorbed (latent heat is released), causing the humidity ratio to drop while the dry-bulb temperature rises sensibly.

Cooling, Dehumidifying, & Reheating Systems

In tropical zones or highly crowded institutional spaces, ambient outside air frequently features an acceptable temperature but carries an excessively high humidity ratio. Introducing this air directly into an occupied zone violates the upper comfort limit (RH > 60%). To treat this air, systems employ a multi-stage process consisting of a **Dehumidifier Section (Cooling Coils)** followed immediately by a **Heating Section (Reheat Coils)**.



The process proceeds through three distinct thermodynamic states:

1. **State 1 to State 2 (Cooling and Dehumidification):** The moist air stream passes over a chilled-water or direct-expansion cooling coil. The surface temperature of this coil is maintained significantly below the dew point of the entering air. As the air cools sensibly, it reaches its dew point, becomes fully saturated (RH = 100%), and further cooling forces water vapor to condense into a liquid state. This liquid water falls out of the air stream at the apparatus dew point temperature (t_2).
2. **State 2 to State 3 (Sensible Reheating):** The saturated, cold air leaves the dehumidifier section. If supplied directly to a room, it would cause localized chilling discomfort. It is passed over a heating coil where heat is added sensibly. This increases its dry-bulb temperature to the final supply condition while its absolute moisture mass remains fixed ($SH_2 = SH_3$). This sensible temperature rise drops the relative humidity into the comfort zone.

A. Mass Balance Equations

Dry air does not condense or accumulate within the system. Therefore, the mass flow rate of dry air (m_a) remains constant across all sections:

$$m_{a1} = m_{a2} = m_{a3} = m_a$$

The mass flow rate of dry air is calculated from the volumetric flow rate at the inlet (V_1) divided by the specific volume of the moist air mixture (v_1) at that exact state:

$$m_a = \frac{V_1}{v_1}$$

The mass flow rate of the water vapor changes as condensation occurs. The mass balance of water vapor inside the dehumidifier section yields:

$$m_{v1} = m_w + m_{v2}$$

By expressing the water vapor mass as a function of the Specific Humidity (SH, or humidity ratio ω), where $m_v = SHm_a$, we isolate the mass rate of the condensed water (m_w):

$$m_w = m_{v1} - m_{v2}$$

$$m_w = m_a\left(SH_1 - SH_2\right)$$

Energy Balance: Dehumidifier Section

Applying the steady-flow energy equation to the cooling coil section ($E_{IN} = E_{OUT}$), we note that energy enters via the moist air stream at State 1, and energy leaves via the cooled air stream at State 2, the liquid condensate at State 4, and the heat removed by the cooling medium (Q_L):

$$m_a h_1 = Q_L + m_a h_2 - m_w h_4$$

Substituting the mass balance relation for the condensate water (m_w) into this energy balance:

$$m_a h_1 = Q_L + m_a h_2 - m_a\left(SH_1 - SH_2\right)h_4$$

Isolating the cooling load capacity (Q_L):

$$Q_L = m_a\left(h_1 - h_2\right) - m_a\left(SH_1 - SH_2\right)h_4$$

The enthalpy of the departing condensate (h_4) is determined using compressed liquid or saturated water tables evaluated specifically at the low intermediate dew point temperature (t_2) $h_4 = h_f$ @ t_2

C. Energy Balance: Reheating Section

The reheating coil performs pure sensible heating. No moisture is added or removed $\left(SH_1 = SH_3\right)$. The energy balance across this control volume simplifies to:

$$m_a h_2 + Q_{heat} = m_a h_3$$

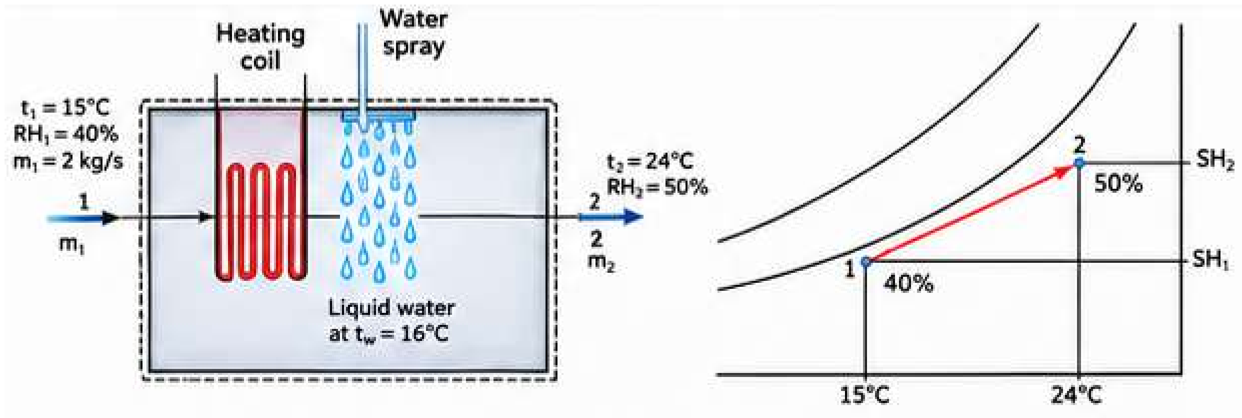
$$Q_{heat} = m_a\left(h_3 - h_2\right)$$

Using the specific heat capacity of dry air (C_{Pa} = 1.005 kJ/kg-°C) paired with a corrected vapor factor, this sensible energy addition can also be closely approximated by:

$$Q_{heat} = m_a C_{pa}\left(T_3 - T_2\right)$$

Heating and Humidifying

The heating and humidifying process is critically applied in cold-climate HVAC systems or specialized industrial processing spaces where ambient winter air features both an unacceptably low temperature and low absolute moisture content. If dry cold air is introduced directly into a space, it causes rapid skin moisture evaporation and respiratory discomfort. To remedy this, the air conditioning system first routes the air across a sensible heating coil to raise its temperature, followed by a chamber where a liquid water spray or steam jet actively injects moisture.



Let the intermediate state between the heater and the water spray section be designated as State i.

1. Across the sensible heating coil (State 1 to State i), no moisture is added, so $SH_1 - SH_i$. The heat input rate is simply:

   $$Q = m_a\left(h_i - h_1\right)$$

2. Across the liquid water spray chamber (State i to State 2), no external heat is added $\left(Q_{spray} = 0\right)$. Liquid water at temperature t_w enters the system at mass flow rate m_w and changes the air state from i to 2.

Applying a total mass and energy balance across the water spray chamber gives:

Mass Balance of water: $m_w = m_a\left(SH_2 - SH_i\right) = m_a\left(SH_2 - SH_1\right)$

Energy Balance: $$m_a h_i + m_w h_w = m_a h_2$$

$$m_a h_i = m_a h_2 - m_w h_w$$

Now, substituting this expression for $m_a h_i$ back into our heating coil work equation yields the correct total energy equation:

$$Q = \left(m_a h_2 - m_w h_w\right) - m_a h_1$$

$$Q = m_a\left(h_2 - h_1\right) - m_w h_w$$

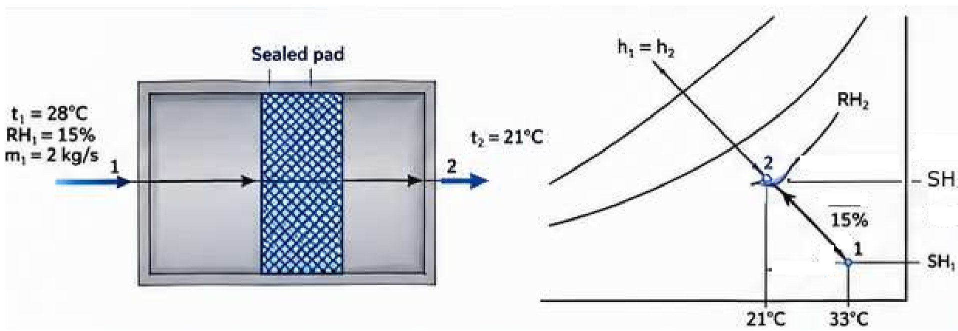
Substituting the mass relationship for m_w reveals the true governing system equation:

$$Q = m_a\left(h_2 - h_1\right) - m_a\left(SH_2 - SH_1\right)h_w$$

The term $-m_a(SH_2 - SH_1)h_w$ accounts for the internal enthalpy carried into the air stream by the water spray itself. This means the external heating coil does not need to supply this energy to reach the final state enthalpy (h_2).

Evaporative Cooling

Evaporative cooling is an adiabatic treatment process deployed in hot, arid climates where the ambient air temperature is high but relative humidity is exceptionally low. The air stream passes through a thick, porous pad or matrix that is constantly soaked with water. Because the air is dry, it has a high capacity to absorb moisture. As the unsaturated air contacts the wet surface, water evaporates directly into the air stream.



The thermal energy required to vaporize the liquid water comes directly from the sensible heat store of the passing air stream. As a result, the dry-bulb temperature of the air drops while its humidity ratio increases.

Because no external heat energy is added or extracted from the overall system, this process is classified as adiabatic. On a psychrometric chart, the state vector tracks upward and to the left along a line of **constant enthalpy** ($h_1 = h_2$) and **constant wet-bulb temperature**. The theoretical limit of this cooling process is the wet-bulb temperature of the entering air.

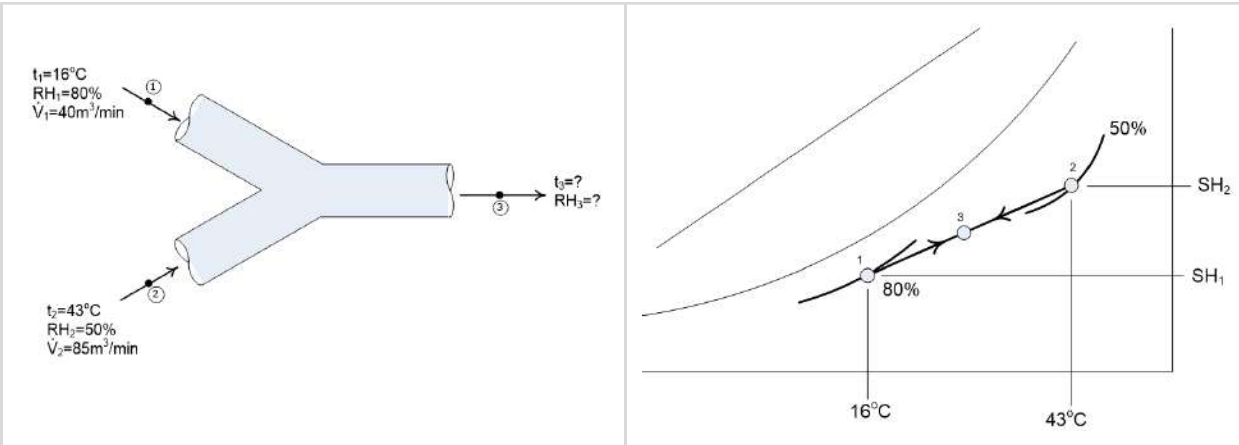
Mass Balance Analysis

The mass flow rate of dry air remains constant ($m_{a1} = m_{a2} = m_a$). The amount of liquid water added to the air stream per unit time (m_w) is governed by the change in specific humidity across the pad:

$$m_w = m_a(SH_2 - SH_1)$$

Adiabatic Mixing of Air Streams

In commercial air conditioning systems, it is inefficient to continuously cool 100% fresh outside air. Instead, conditioned air returning from the room is mixed with a regulated volume of fresh outdoor ventilation air before being routed back through the main cooling coils.



When two distinct moist air streams merge under steady-flow conditions without any external heat transfer or work interactions, the process is called **adiabatic mixing**.

A. Conservation Equations

- **Dry Air Mass Conservation:**

$$m_{a1} + m_{a2} = m_{a3}$$

- **Water Vapor Mass Conservation:**

$$m_{a1}SH_1 + m_{a2}SH_2 = (m_{a1} + m_{a2})SH_3$$

- **Total Energy Conservation:**

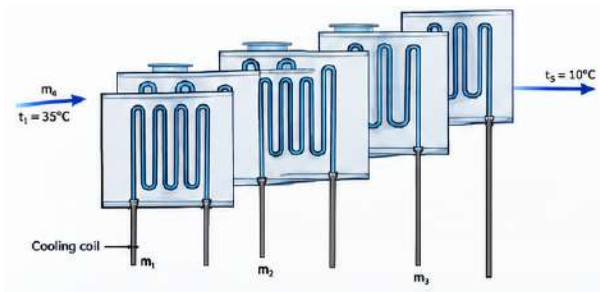
$$m_{a1}h_1 + m_{a2}h_2 = (m_{a1} + m_{a2})h_3$$

On the psychrometric chart, the final mixture state (State 3) will always lie exactly on a straight line connecting the two initial operating points (State 1 and State 2). The relative position of State 3 along this line acts like a balance scale, proportional to the mass flow rate fractions of the constituent streams:

$$\frac{m_{a1}}{m_{a2}} = \frac{h_2 - h_3}{h_3 - h_1} = \frac{SH_2 - SH_3}{SH_3 - SH_1} = \frac{t_{d2} - t_{d3}}{t_{d3} - t_{d1}}$$

Contact Factor (CF) vs. Bypass Factor (BF)

When air passes through a physical cooling or heating coil, the heat exchange is not perfectly uniform. A portion of the air stream makes direct physical contact with the metal fin surfaces and is fully conditioned to the apparatus dew point temperature (t_{adt}). The remaining fraction passes through the open spacing between the tubes and fins without interacting with the cold surfaces at all.



- **Bypass Factor (BF):** The ratio of the air mass that bypasses the coil surface (m_B) to the total passing air mass (m_T).

$$BF = \frac{m_B}{m_T}$$

- **Contact Factor (CF):** The ratio of the air mass that directly contacts the coil surface (m_C) to the total passing air mass (m_T).

$$CF = \frac{m_C}{m_T}$$

Because all entering air must either contact the surface or bypass it, these two factors complement each other perfectly:

CF + BF = 1

Process Properties

These performance factors can be calculated using temperatures, specific humidities, or enthalpies derived from the psychrometric chart:

$$CF = \frac{t_i - t_o}{t_i - t_{adt}} = \frac{SH_i - SH_o}{SH_i - SH_{adt}} = \frac{h_i - h_o}{h_i - h_{adt}}$$

$$BF = \frac{t_o - t_{adt}}{t_i - t_{adt}} = \frac{SH_o - SH_{adt}}{SH_i - SH_{adt}} = \frac{h_o - h_{adt}}{h_i - h_{adt}}$$

Where:

- t_i , SH_i, h_i = Properties of the air entering the coil.
- t_o , SH_o, h_o = Bulk average properties of the air exiting the coil.
- t_{adt} , SH_{adt}, h_{adt} = Properties evaluated at the apparatus dew point temperature (the temperature of the wet coil surface itself).

Multi-Row Coil Scaling Law: If an operating system stacks multiple identical cooling coil rows in series, the net cumulative bypass factor of an n-row assembly is found by raising the bypass factor of a single row to the power of n:

$$BF_{net} = \left(BF_{single\ row}\right)^n$$

Air-Conditioning Load

To accurately size air conditioning equipment, engineers divide the total thermal energy entering a space (Q_T) into two components based on how the heat affects the air properties.

Sensible Heat Load (Q_S)

Sensible heat is the thermal energy associated with changes in the dry-bulb temperature of a substance without any phase change. In a building, sensible loads are driven by solar radiation through glass, conduction through walls, and the heat emitted by lights, computers, and occupants.

$$Q_S = m_S C_{pa}\left(t_2 - t_1\right)$$

Where C_{pa} is the specific heat capacity of dry air, and $\left(t_2 - t_1\right)$ is the dry-bulb temperature differential.

Latent Heat Load (Q_L)

Latent heat is the thermal energy associated with changes in the moisture content of the air through phase changes (evaporation or condensation) while the dry-bulb temperature remains constant. Latent loads are driven by outdoor air infiltration, occupant respiration, and moisture-generating processes like cooking or washing.

$$Q_L = m_S\left(SH_2 - SH_1\right)h_v$$

Where h_v represents the latent heat of vaporization for water vapor at standard indoor design conditions, commonly taken as a constant baseline:

h_v = 2,442 kJ/kg

Total Heat Load (Q_T) and the Sensible Heat Factor (SHF)

The total cooling load that the air conditioner must remove is the sum of both components:

$$Q_T = Q_S + Q_L$$

The ratio of sensible cooling capacity to the total cooling capacity is called the **Sensible Heat Factor (SHF)** or **Sensible Heat Ratio (SHR)**:

$$SHF = \frac{Q_S}{Q_S + Q_L}$$

On a psychrometric chart, the slope of the process line matches this ratio. Designing a system requires matching the coil's operational slope to the room's calculated SHF line to ensure both temperature and humidity remain stable.

Cooling Tower

A cooling tower is a specialized industrial heat exchanger designed to reject waste heat from a facility by cooling a circulating water stream to a lower temperature and transferring that thermal energy directly to the atmosphere. In large-scale power generation facilities, chemical plants, and commercial central HVAC installations, massive quantities of condenser waste heat must be continuously expelled to maintain structural thermodynamic cycles. Rather than continuously consuming fresh water via open, once-through configurations, a cooling tower enables a highly efficient recirculating loop by leveraging the atmosphere as an ultimate heat sink.

Evaporative Cooling Mechanisms

The primary phenomenon driving heat rejection inside a cooling tower is **evaporative cooling**. As warm water enters the top of the tower, it is distributed across internal grids and broken down into fine droplets or thin films to maximize surface area. Simultaneously, a moving stream of unsaturated ambient air passes over this cascading water.

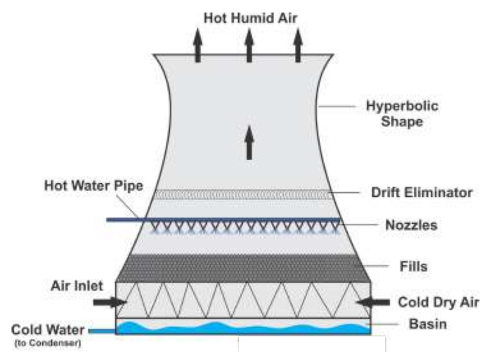
Because the vapor pressure at the surface of the warm water droplets exceeds the partial pressure of water vapor in the passing air stream, a small fraction of the circulating water evaporates directly into the air. The latent heat of vaporization required for this phase change is drawn directly from the remaining liquid water stream, causing its sensible temperature to drop sharply. Approximately 1% to 2% of the total water flow is deliberately vaporized to cool the remaining 98% to 99% of the stream.

Classification of Cooling Towers

Cooling towers are structurally classified based on the mechanism utilized to move the atmospheric air through the falling water matrix.

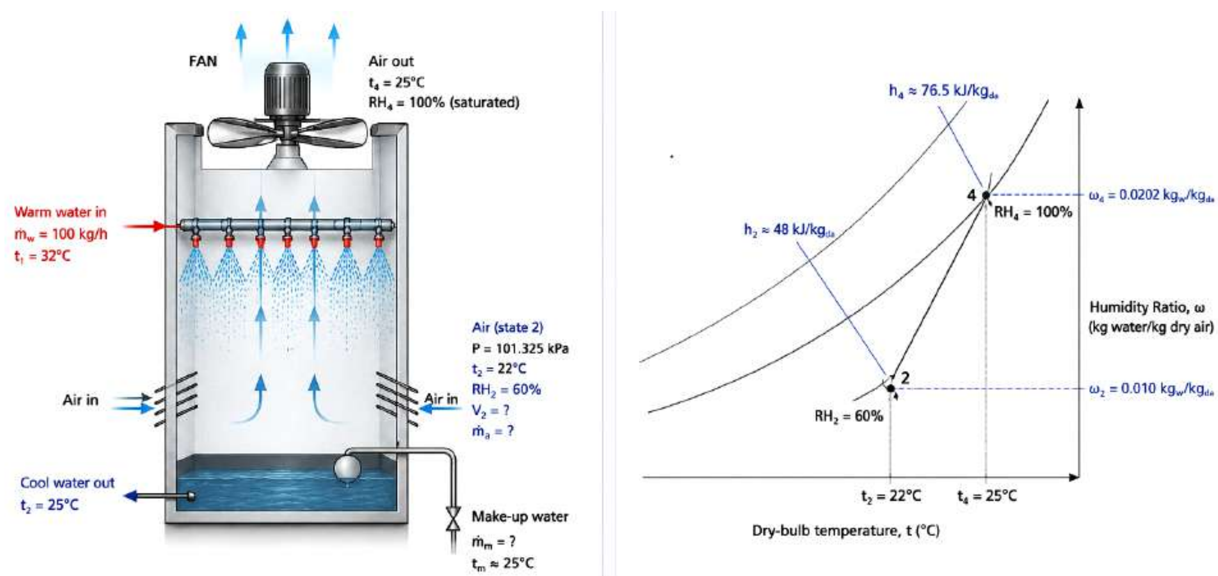
Natural Draft Cooling Towers

Natural draft cooling towers rely entirely on the buoyancy effects of a chimney stack to establish air movement. These towers feature iconic, tall hyperbolic concrete shells. As the air inside the tower absorbs heat and moisture from the hot water, its density decreases compared to the cold, dense ambient air outside. This density difference generates a natural draft that draws fresh air through the bottom intake and pushes warm, humid air out the top without requiring mechanical fans. They have high initial capital costs but exceptionally low operating electricity requirements.



Mechanical Draft Cooling Towers

Mechanical draft cooling towers use power-driven electric fans to force or pull a controlled volume of air through the internal structure. This approach provides highly stable, predictable cooling performance regardless of shifting weather conditions. Mechanical draft systems are subdivided into two geometric configurations:



- **Induced Draft Cooling Towers:** The mechanical fan is located at the top air discharge outlet. The fan pulls air upward through the tower packing, creating a vacuum that draws fresh air through intake louvers at the base. This design ensures a high exit velocity, preventing recirculation of warm discharge air back into the air intakes.
- **Forced Draft Cooling Towers:** The mechanical fan is located at the fresh air inlet base. The fan forces ambient air into the tower structure, building a positive static pressure that drives the air upward through the internal packing matrix and out the top discharge.

Mass Conservation Formulation

Dry air behaves as a non-condensable carrier gas. Therefore, the mass flow rate of dry air (m_a) entering the base louvers matches the mass flow rate leaving the top fan discharge:

$$m_a = m_b$$

The absolute mass of the water inside the system undergoes a continuous shift because liquid water drops are transformed into vapor. The overall water mass balance across the tower envelope is formulated as:

$$m_{w1} + m_{va} + m_M = m_{w2} + m_{vb}$$

Where:

- m_{w1} = Mass flow rate of incoming warm water.
- m_{w2} = Mass flow rate of outgoing cooled water.
- m_{va}, m_{vb} = Mass flow rates of the water vapor suspended in the air at the inlet and exit.
- m_M = Mass flow rate of required make-up water.

By design, the mass flow rate of dry air is constant, and the circulating water loop is continuous $\left(m_{w1} = m_{w2} = m_w\right)$. Using specific humidity $SH = \frac{m_v}{m_a}$, we track the rate of moisture accumulation in the passing air stream to isolate the required mass flow rate of make-up water $\left(m_M\right)$:

$$m_M = m_{vb} - m_{va}$$

$$m_M = m_a\left(SH_b - SH_a\right)$$

First Law Energy Balance

Applying the steady-flow energy equation ($E_{IN} = E_{OUT}$) across the boundary of the cooling tower yields:

$$m_w h_1 + m_a h_a + m_M h_M = m_w h_2 + m_a h_b$$

Substituting our mass balance relation for the make-up water $m_M = m_a(SH_b - SH_a)$ directly into the energy equation gives:

$$m_w h_1 + m_a h_a + m_a(SH_b - SH_a)h_M = m_w h_2 + m_a h_b$$

Finding the required mass flow rate of dry air (m_a)

$$m_a = \frac{m_w(h_1 - h_2)}{(h_b - h_a) - (SH_b - SH_a)h_M}$$

If the specific temperature of the make-up water supply is not explicitly stated in a board problem, it is an industry-standard convention to assume it enters at the final basin temperature:

$$h_M = h_2 = h_f \ @ \ t_2$$

Once mass flow rates are calculated, you can convert them into real-world volumetric flow rates (V) to select the appropriate pump and fan sizes using specific volume (v) parameters:

- **Air Fan Flow Intake:** $V_a = m_a v_a$
- **Water Pump Loop:** $V_1 = m_w v_1$

$$\text{Where: } v_1 = v_f \ @ \ t_{ave} = v_f \ @ \ \frac{t_1 + t_2}{2}$$

To evaluate the efficiency of a cooling tower, engineers track three main parameters: **Range**, **Approach**, and **Cooling Tower Efficiency (CTE)**.

Cooling Range (R)

The cooling range is the difference between the temperature of the warm water entering the tower and the temperature of the cooled water leaving the tower. It indicates the total sensible heat load delivered to the cooling tower from the facility's condenser loops.

$$R = t_1 - t_2$$

Approach Temperature (A)

The approach is the difference between the temperature of the cooled water leaving the tower and the wet-bulb temperature (t_{wa}) of the ambient air entering the tower. The ambient wet-bulb temperature represents the absolute thermodynamic limit for evaporative cooling. A cooling tower cannot cool the circulating water below this value. Therefore, the approach temperature is a direct indicator of how closely a tower's physical design matches its ideal thermodynamic limit.

$$A = t_2 - t_{wa}$$

Cooling Tower Efficiency (CTE)

Cooling tower efficiency is the ratio of the actual water temperature drop achieved to the maximum ideal temperature drop that would be possible in an infinitely large tower (where the leaving water temperature drops all the way to the ambient wet-bulb temperature). It is expressed mathematically as:

$$CTE = \frac{Actual\ Range}{Ideal\ Range} = \frac{R}{R+A} = \frac{t_1 - t_2}{t_1 - t_{wa}}\ x\ 100\%$$

Dryers & Mechanical Drying Systems

A dryer is a piece of thermal machinery designed to remove relatively small amounts of water or volatile moisture from a solid product. While processes like evaporation typically deal with concentrating liquid solutions, drying targets solid or semi-solid feeds (such as agricultural grains, paper pulp, minerals, and chemical powders), reducing their moisture content to an acceptable preservation or processing threshold. In industrial engineering applications, the structural moisture inside a material must be managed carefully to avoid biological degradation, prevent corrosion, or satisfy strict product quality standards.

When conducting mass balance calculations for a solid material undergoing drying, it is critical to split the total mass (m_T) into two distinct components:

- **Mass Bone-Dry (m_{BD}):** The absolute, solid portion of the material containing no moisture. This matrix component is completely non-volatile and remains perfectly constant before, during, and after the thermal drying process ($m_{BD,\ in} = m_{BD,\ out}$).
- **Mass of Moisture (m_M):** The total mass of water or fluid physically trapped or chemically bound within the solid product. This value actively decreases as vapor is driven off into the surrounding air stream.

The total mass of the product at any given point is the sum of these two values:

$$m_T = m_{BD} + m_M$$

To express the moisture presence in a way that matches standard engineering problems, two distinct mathematical bases are used:

A. Moisture Content on a Wet-Basis (x_M or y)

The wet-basis moisture content expresses the mass of liquid moisture as a fraction of the total mass of the wet product. This metric is widely used in commercial commerce and product specifications.

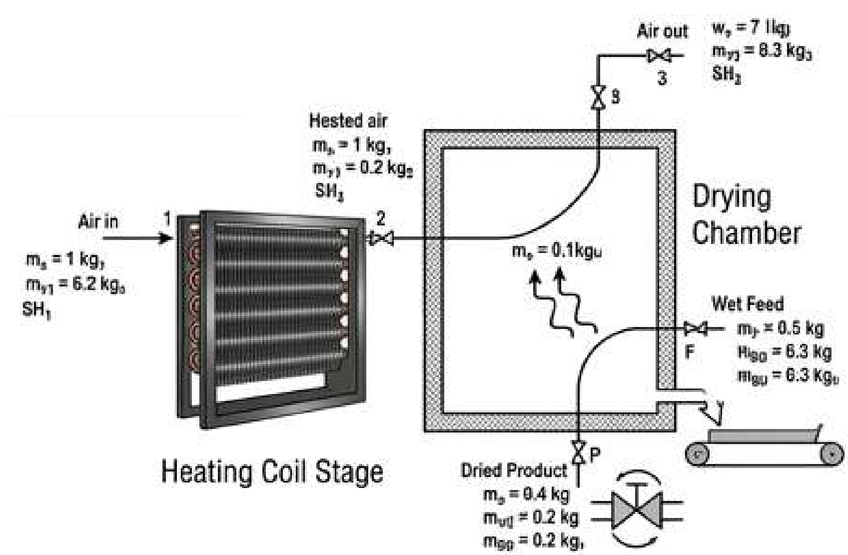
$$x_M = \frac{m_M}{m_T} = \frac{m_M}{m_{BD} + m_M}$$

B. Moisture Content on a Dry-Basis (P_M or X)

The dry-basis moisture content (sometimes referred to as the moisture regain ratio) expresses the mass of liquid moisture as a fraction of the constant bone-dry mass of the solid. Because the denominator stays constant throughout the process, dry-basis values simplify tracking moisture changes during multiple processing stages.

$$P_M = \frac{m_M}{m_{BD}}$$

Mechanical or convective drying is a process where an atmospheric air stream is thermally treated (preheated) to reduce its relative humidity, preparing it to absorb moisture from a wet material. As this high-temperature, unsaturated air passes through the drying chamber, it provides the latent heat needed to vaporize the liquid moisture inside the solid material. The evaporated water is then swept out of the system by the exhaust air stream.



A. Air-Side Vapor Dynamics

The mass flow rate of dry air carrier gas (m_a) remains constant across the preheater coil and the drying chamber. The total mass rate of moisture carried away from the solid material by the air (m_v) matches the change in specific humidity between the inlet and exhaust states:

$$m_v = m_a\left(SH_3 - SH_2\right)$$

Since the process across the preheater coil is purely sensible, the specific humidity entering the drying chamber matches the ambient inlet value $\left(SH_2 - SH_1\right)$). This simplifies the air-side moisture absorption equation to:

$$m_v = m_a\left(SH_3 - SH_1\right)$$

B. Solid-Side Mass Balancing

Let W_F represent the mass flow rate of the incoming wet feed, and let W_P represent the mass flow rate of the exiting dried product. Let y_F and y_p represent their respective wet-basis moisture fractions. By conservation of mass, the moisture removed from the material (m_v) must equal the total mass drop of the product:

$$m_v = W_F - W_P$$

Because the absolute mass flow rate of bone-dry solid is constant, the dry mass balance equation is written as:

$$W_{BD} = \left(1 - y_F\right)W_F = \left(1 - y_P\right)W_P$$

This identity allows you to calculate the output product weight directly from the input feed parameters without needing to compute intermediate air properties:

$$W_P = W_F\left(\frac{1 - y_F}{1 - y_P}\right)$$

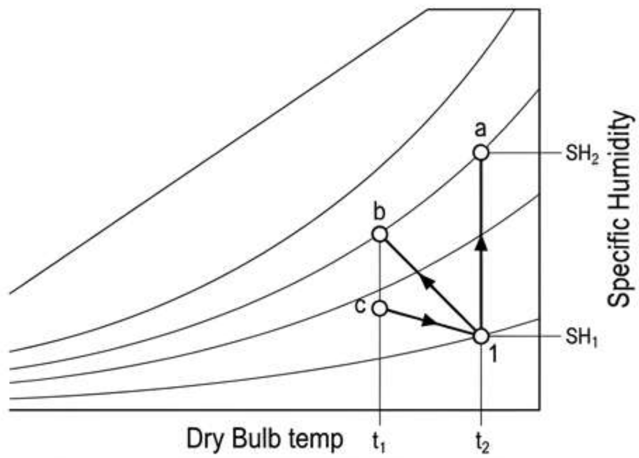
C. Preheater Energy Requirements

The external heat transfer rate (Q) supplied to the air by the preheater coil depends on the dry air mass flow rate and the change in enthalpy across the coil:

$$Q = m_a\left(h_2 - h_1\right)$$

Psychrometric Dryer Classification

The thermodynamic behavior of a drying process can be mapped onto a psychrometric chart. Dryers are classified into three types based on their internal heat transfer conditions:



Isothermal Dryer (1 - a)

An isothermal dryer maintains a constant dry-bulb temperature throughout the drying chamber $\left(t_2 = t_3\right)$. To achieve this, steam coils or reheat panels are installed directly inside the drying bed to supply supplemental sensible heat as evaporation occurs. This prevents the air from cooling, maximizing its capacity to absorb moisture. On the psychrometric chart, this process tracks straight upward as a **pure humidifying vector**. It represents the theoretical maximum drying rate for a given chamber volume.

Adiabatic Dryer (1- b)

In an ideal adiabatic dryer, the drying chamber is perfectly insulated against external heat loss ($Q_{chamber}$ = 0). The entire latent heat of vaporization is supplied by the sensible heat of the preheated entering air. As water evaporates, the dry-bulb temperature of the air drops while its specific humidity increases. This process follows a line of **constant enthalpy** ($h_2 = h_3$) and constant wet-bulb temperature, tracking upward and to the left on the psychrometric chart as a **sensible cooling and humidification vector**.

Non-Adiabatic Dryer (1 - c)

A non-adiabatic dryer represents the operation of an actual physical drying system. Because real drying chambers cannot be perfectly insulated, a portion of the air's sensible heat is lost through the outer walls to the surroundings. Consequently, both the air temperature and the total mixture enthalpy drop more rapidly than in the ideal adiabatic case. The state vector tracks below the constant-enthalpy line, moving downward and to the left as a modified **sensible cooling and humidification path**.