Step 1: Analyze the Given Parameters and Establish Standard Constants

In fluid machinery board exam problems, when standard air properties are not explicitly listed, we use the standard density of air (ρ_a = 0.075 lb/ft^3) and the standard density of water (ρ_W = 62.4 lb/ft^3).

- Volumetric Flow Rate (Q) = 230 ft^3/sec}
- Duct Dimensions = 2.5 ft x 4.5 ft
- Total Pressure Head (h_W) = 4.2 inches of water

Step 2: Convert Total Pressure Head to Total Head in Feet of Air

To determine the power exerted by the fan, the total pressure head given in inches of water must be converted into an equivalent column height of the working fluid itself—in this case, feet of air (H_T). We equate the pressure exerted by the two fluid columns $\rho_a H_T = \rho_W h_W$ and convert inches to feet:

$$H_T = h_W \left(\frac{1\, ft}{12\, in}\right)\left(\frac{\rho_W}{\rho_a}\right)$$

Substituting our values into the fluid balance equation:

$$H_T = (4.2\, in\, of\, water)\left(\frac{1\, ft}{12\, in}\right)\left(\frac{62.4\, lb/ft^3}{0.075\, lb/ft^3}\right) = 291.2\, ft\, of\, air$$

Physically, this tells us that because water is 832 times denser than standard air, it takes a column of air 291.2 feet tall to exert the exact same downward pressure as a tiny column of water only 0.35 feet (4.2 inches) deep.

Step 3: Compute the Air Power (Air Horsepower)

Air Power (AP) represents the ideal, frictionless energy per unit time delivered to the air stream. The standard thermodynamic equation for fluid power is the product of the weight flow rate and the total head:

$$AP = \frac{\gamma_a Q H_T}{550}$$

Where $\gamma_a = \rho_a \left(\frac{g_o}{g_C}\right) = 0.075 \frac{lb_f}{ft^3}$, and 550 ft-lb/sec-hp is the mechanical conversion factor for horsepower. Substituting the parameters yields:

$$AP = \frac{(0.075\, lb/ft^3)(230\, ft^3/s)(291.2\, ft\, of\, air)}{550}$$

AP = 9.133 hp

The dimensions of the duct (2.5ft x 4.5 ft) allow us to determine that the air is moving at a velocity of 20.44 ft/s. In ventilation system design, understanding the relationship between volume flow rate and duct restriction is vital.

A fan overcomes two things: **Static Pressure** (the friction of the air rubbing against the duct walls and dampers) and **Velocity Pressure** (the kinetic energy needed to accelerate the mass of the air). Total pressure combines both.

The Air Horsepower computed here (9.13 hp) is the purely theoretical *ideal* power. In actual plant environments, fans operate at mechanical efficiencies (η_f) hovering around 60% to 80%. An engineer reviewing this system would know that to safely deliver this fluid performance, the actual brake horsepower $\left(BHP = \frac{AP}{\eta_f}\right)$ required from the electric motor would need to be scaled up to roughly a standard **15 hp** motor.