Step 1: Analyze the Given Parameters

Before substituting values into equations, we must systematically list all known values and establish the underlying thermodynamic assumptions.

- Suction Pressure (P_1) = 101.325 kPa
- Discharge Pressure (P_2) = 680 kPa
- Volumetric Flow Rate ($V_{1'}$) = 0.35 m^3/s
- Isentropic Exponent (k) = 1.4 (standard for air)

Step 2: Calculate the Work Required for a Single-Stage Compressor

For a single-stage, uncooled compressor, the steady-flow work required to compress an ideal gas is isentropically is given by the standard formula:

$$W_{C,1} = \frac{kP_1V_{1'}}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1\right]$$

Substituting the given values into the equation:

$$W_{C,1} = \frac{(1.4)(101.325\ kPa)(0.35m^3/s)}{1.4-1}\left[\left(\frac{680\ kPa}{101.325\ kPa}\right)^{\frac{1.4-1}{1.4}} - 1\right] = 89.7120\ kW$$

Step 3: Determine the Optimum Intermediate Pressure for Two Staging

To minimize total power consumption across two stages with perfect intercooling, the pressure ratio across each stage must be identical. This occurs when the intermediate pressure (P_x) is the geometric mean of the suction and final discharge pressures:

$$P_X = \sqrt{P_1P_2} = \sqrt{(101.325\ kPa)(680\ kPa)} = 256.4899\ kPa$$

Step 4: Calculate the Work Required for a Two-Stage Compressor

With perfect intercooling, the gas is cooled back to its initial suction temperature before entering the second stage. The total ideal work for a two-stage system is twice the work of a single stage operating under the optimum stage pressure ratio:

$$W_{C,1-2} = \frac{2kP_1V_{1'}}{k-1}\left[\left(\frac{P_x}{P_1}\right)^{\frac{k-1}{k}} - 1\right]$$

Substituting the optimized intermediate pressure:

$$W_{C,1-2} = \frac{2(1.4)(101.325\ kPa)(0.35m^3/s)}{1.4-1}\left[\left(\frac{256.4899\ kPa}{101.325\ kPa}\right)^{\frac{1.4-1}{1.4}} - 1\right] = 75.4417\ kW$$

<u>Step 5:</u> Calculate the Total Power Saved

The power saved by upgrading from a single-stage to a two-stage compressed system is simply the difference between their respective work requirements:

Power Savings = $W_{C,1} - W_{C,1-2}$

Power Savings = $89.7120\ kW - 75.4417\ kW$

Power Savings = $14.2\ kW$

When gas is compressed in a single stage without cooling, its temperature rises drastically, expanding the gas and forcing the piston to push against a larger volume. This increases the total work requirement. By breaking the compression into two stages and dropping the temperature back down mid-way using an intercooler, we significantly reduce the specific volume of the gas entering the second stage.

Beyond saving approximately **16%** of power in this specific scenario, staging lowers the peak discharge temperature. This prevents the degradation of compressor lubricants, reduces thermal stress on mechanical components, and prevents volatile oil vapors from combusting inside the discharge lines.