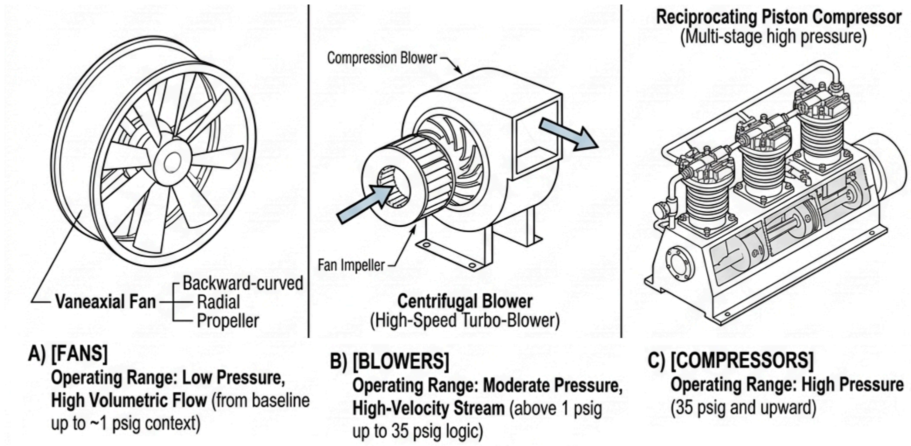
In industrial process plants and HVAC installations, the movement of air or gases under moderate pressures requires fluid machines categorized as fans or blowers. These machines serve as the primary movers for ventilation, combustion air supply, and pneumatic conveying systems. Understanding their mechanical classifications, operational parameters, and thermodynamic energy dynamics is a staple of mechanical engineering board exams.



Fluid moving machines are distinctly classified based on their operating discharge pressure levels. It is critical for the exam candidate to remember the standard boundary values that distinguish fans, blowers, and compressors:

- **Fans:** These are turbomachines that move air or gases at very low pressures, generally operating from a baseline up to approximately 1 psig (6.89 kPa). They are designed for high volumetric flow rates with minimal pressure elevation.
- **Blowers (Turbo-blowers):** These machines operate at moderate pressures, spanning from above 1 psig up to a strict limit of 35 psig (241.3 kPa). Blowers deliver a more concentrated, high-velocity gas stream compared to fans.
- **Compressors:** Any gas-moving machine designed to discharge gas at pressures starting from 35 psig and extending upward into high-pressure applications falls under the category of a compressor.

FANS

Fans are further sub-classified according to the direction of gas flow through the impeller mechanism:

1. **Centrifugal Fans:** These units move air or gases radially through the impellers. The gas is drawn axially into the center of a revolving wheel (connected to a rotating shaft) and is turned 90^{o} as it enters the spaces between the wheel blades, accelerating centrifugally into the fan housing before discharging. Centrifugal fans are subdivided based on blade geometry:
    - **Forward-curved:** Blades curve in the direction of rotation; suited for low-pressure, high-volume operations.
    - **Backward-curved:** Blades curve away from the direction of rotation; quieter and highly efficient with non-overloading power characteristics.
    - **Radial:** Straight blades extending out from the hub; highly robust and ideal for industrial material handling containing dust or debris.

2. **Axial Fans:** These machines move the gas stream parallel to the axis of the rotating shaft. The air enters and exits along a straight path without changing radial direction. They are categorized as:
    - **Propeller:** Basic, low-pressure configurations typically used for free-delivery ventilation (e.g., wall exhaust fans).
    - **Tubeaxial:** A propeller fan enclosed within a cylindrical tube wrapper; offers higher pressure capabilities and improved flow direction.
    - **Vaneaxial:** A tubeaxial fan equipped with air-guide vanes positioned either before or after the wheel; maximizes static pressure efficiency and streamlines the discharge profile.

Air Power (AP) represents the useful thermodynamic energy added by the fan impeller to the gas stream to overcome the system resistance at a given flow rate. Depending on which pressure head component is utilized, engineers calculate either **Total Air Power** or **Static Air Power**.

Total Air Power Calculation

When evaluating the complete energy state change of the fluid, the **Total Pressure Head** (H_T) is applied. This encompasses both the static pressure (resistance to flow) and the velocity pressure (kinetic energy of the gas).

The fundamental equation expressed in volumetric flow terms is:

$$AP = Q\gamma_a H_T$$

Where:

- AP = Total air power, measured in kilowatts (kW) or horsepower (hp). *(Conversion factor: 1 hp = 0.746kW)*
- Q (or V) = Volumetric flow rate, measured in m^3/s or ft^3/sec.
- H_T = Total pressure head of the fan, measured in meters or feet of the fluid (air) column.
- γ_a = Specific weight of air.

Specific Weight Transformations and Dimensional Adaptations

To establish the specific weight of the gas stream (γ_a), the material relates density (ρ_a) to local gravitational constants using standard mass-to-weight formatting:

$$\gamma_a = \rho_a\left(\frac{g_o}{g_c}\right) = \frac{m_a}{V_a}\left(\frac{g_o}{g_c}\right)$$

Where:

- γ_a = Density of air (kg/m^3 or lbm/ft^3).
- m_a = Mass flow rate of air (kg/s or lbm/sec).
- g_o = Local acceleration due to gravity.
- g_C = Gravitational conversion factor (dimensionless in SI, but equal to 32.2lbm-ft/lbf-sec^2 in English systems).

Standard Air Condition Properties:

For ambient dry air at a standard temperature of **21ºC** (70ºF) and a barometric pressure of **101.325 kPa** (14.7 psi):

- **SI Units Specific Weight (γ_a):** 0.0118 kN/m^3 *(Note: Equivalent to a standard density ρ_a = 1.20 kg/*m^3 *multiplied by local gravity)*
- **English Units Specific Weight (γ_a):** 0.075 lbf/ft^3

Substituting the mass relationship into the volumetric equation yields the alternative mass flow rate expression for Total Air Power:

$$AP = m_a\left(\frac{g_o}{g_c}\right)H_T$$

Static Air Power Calculation

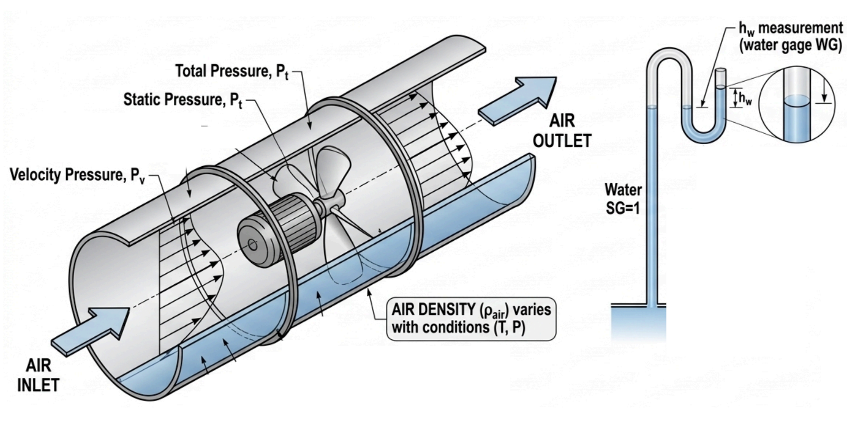
In many engineering systems, the velocity energy at the discharge is lost to the atmosphere, meaning only the static component does useful work. In these cases, the **Static Air Power** (AP_{static}) is evaluated by substituting the **Static Pressure Head** (h_s) in place of the total head:

$$AP_{static} = Q\gamma_a h_s$$

Similarly, expressed as a function of the mass flow rate:

$$AP_{static} = m_a\left(\frac{g_o}{g_c}\right)h_s$$

Where h_s represents the static pressure head of the fan measured in meters of air or feet of air.



First Law Derivation for Fan Power (AP)

To establish a comprehensive mathematical framework, we apply the General Energy Equation (Conservation of Energy) to a steady-flow fan system. The energy entering the system boundaries must balance perfectly with the energy exiting the boundaries:

$$E_{IN} = E_{OUT}$$

$$PE_1 + KE_1 + Wf_1 + U_1 + AP = PE_2 + KE_2 + Wf_2 + U_2$$

Where:

- PE represents the potential energy of the gas column.
- KE represents the kinetic energy of the gas flow stream.
- Wf represents the flow work (PV) required to move the fluid across the boundaries.
- U represents the internal thermal energy of the gas.
- AP represents the Air Power (mechanical energy added directly by the fan impeller).

Isolating the Air Power (AP) yields the general energy difference equation

$$AP = (PE_2 - PE_1) + (KE_2 - KE_1) + (Wf_2 - Wf_1) + (U_2 - U_1)$$

Simplifying Engineering Assumptions

For typical low-pressure industrial fans and blowers, the changes in system elevation and temperature profile are so small that specific terms can be omitted:

1. **Negligible Elevation Change:** The suction and discharge duct heights are practically equal ($z_1 = z_2$), meaning potential energy changes are negligible ($\Delta PE = 0$).
2. **Negligible Temperature Change:** The gas experiences little to no temperature change across the fan stage ($T_1 = T_2$), meaning internal energy changes are also negligible ($\Delta U = 0$).

Applying these simplifications reduces the air power expression to the kinetic energy change and flow work change:

$$AP = \frac{1}{2}\frac{m_a}{g_c}\left(v_2^{\,2} - v_1^{\,2}\right) + V_a\left(P_2 - P_1\right)$$

Total Fan Head (H_T)

To convert this energy rate equation (kW or hp) into a linear fluid column height or head (H_T), we multiply both sides of the equation by the mass-weight normalization factor $\frac{g_c}{m_a g_o}$:

$$\left(\frac{g_c}{m_a g_o}\right)AP = \frac{1}{2}\frac{\left(v_2^{\,2} - v_1^{\,2}\right)}{g_o} + \frac{V_a\left(P_2 - P_1\right)}{\frac{m_a g_o}{g_c}}$$

Since specific weight is defined as $\gamma_a = \rho_a\left(\frac{g_o}{g_c}\right) = \frac{m_a}{V_a}\left(\frac{g_o}{g_c}\right)$, substituting this term simplifies the second expression on the right side of the equation:

$$H_T = \frac{1}{2}\frac{\left(v_2^{\,2} - v_1^{\,2}\right)}{g_o} + \frac{\left(P_2 - P_1\right)}{\gamma_a}$$

This expression confirms the relationship defined in our previous module: Total Fan Pressure Head (H_T) is the sum of the system's velocity head (h_V) and static pressure head (h_s):

$$H_T = h_s + h_v$$

Velocity Head (h_v)

The velocity head represents the kinetic energy possessed by the air stream solely due to its linear velocity vector. It measures the energy required to accelerate the gas from its initial intake velocity (v_1) to its higher discharge velocity (v_2):

$$h_v = \frac{1}{2}\frac{\left(v_2^{\,2} - v_1^{\,2}\right)}{g_o}$$

Static Pressure Head (h_s)

The static pressure head represents the energy required by the fan to overcome the internal mechanical resistance of the system.

$$h_s = \frac{\left(P_2 - P_1\right)}{\gamma_a}$$

This head component becomes zero if a fan operates with completely open inlets and outlets without any connected ductwork or dampers. In those unconfined setups, all input power is converted into velocity head.

When evaluating a U-tube water manometer attached to a duct, the static pressure difference $\left(P_2 - P_1\right)$ can be expressed in terms of the balancing column of water (h_w):

$$h_s = \frac{\left(P_2 - P_1\right)}{\gamma_a} = \frac{\gamma_w h_w}{\gamma_a}$$

This yields the two standard board exam head conversion formats used for specific weight and density ratios:

$$h_s = h_w\left(\frac{\gamma_w}{\gamma_a}\right) = h_w\left(\frac{\rho_w}{\rho_{air}}\right)$$

Board exam problems rarely give the fan head directly in "meters of air." They usually give it as the height of a water column (h_w, e.g., in mm of water) using a manometer. Always convert the water head to an equivalent air head using the fluid balance equation:

Fan Efficiencies

The overall performance of a fan is measured by its efficiency, which represents the ratio of the useful air power output to the mechanical brake power input. Brake Power (BP or BHP) is the actual mechanical power supplied to the fan shaft by the driving motor. It is always larger than the net air power due to aerodynamic, friction, and volumetric losses within the housing.

Depending on how the system's output energy is evaluated, manufacturers and engineers define fan efficiency in two ways:

A. Fan Total Efficiency (η_F)

Fan total efficiency (sometimes called mechanical efficiency) represents the machine's ability to convert shaft work into total fluid energy (including both static pressure and kinetic velocity energy). It is used when the kinetic energy of the air leaving the discharge is utilized by downstream duct components:

$$\eta_F = \frac{AP}{BP} \; x \; 100\% = \frac{Q\gamma_a H_T}{BP} \; x \; 100\%$$

Where:

- AP = Total air power
- BP = Brake power input to the fan shaft
- Q = Volumetric flow rate (capacity)
- γ_a = Specific weight of air
- H_T = Total pressure head

B. Fan Static Efficiency ($\eta_{Fstatic}$)

Fan static efficiency evaluates the machine's ability to convert shaft work into useful static pressure head, completely discounting the kinetic energy of the discharge stream. This efficiency parameter is used in systems where the air is discharged directly into a large room or the atmosphere, rendering the velocity pressure lost and unrecoverable:

$$\eta_{Fstatic} = \frac{AP_{static}}{BP} \; x \; 100\% = \frac{Q\gamma_a h_s}{BP} \; x \; 100\%$$

Where:

- AP_{static} = Static air power
- h_s = Static pressure head

Fan Specific Speed (N_s)

The specific speed of a fan is a dimensionless index (though expressed in practical units of rpm) used to classify fan impellers based on their geometric proportions rather than their physical size. It is defined as the rotational speed (in rpm) at which a geometrically similar, scaled-down impeller would operate to deliver a standard rated capacity of 1 cubic foot per minute (cfm) of air against a differential static head of 1 foot of air under standard conditions.

The mathematical expression for fan specific speed is:

$$N_s = \frac{N\sqrt{Q}}{h_s^{3/4}}$$

Where:

- N_s = Specific speed of the fan (rpm)
- N = Impeller rotational speed (rpm)
- Q = Capacity of the fan, measured in cubic feet per minute (cfm)
- h_s = Static pressure head, measured in feet of air

Scaling Adjustments for Multi-Stage and Double-Inlet Systems

When evaluating modified fan configurations, specific parameter adjustments must be made before applying the specific speed formula:

1. **Double-Inlet Fans:** For fans that draw air from both sides of the housing simultaneously (analogous to double-suction pumps), the total design capacity Q must be divided by 2:

$$Q_{design} = \frac{Q_{total}}{2}$$

2. **Multi-Stage Assemblies:** If multiple impellers are configured in series to boost the pressure, the total static head H must be divided by the number of individual stages:

$$H_{design} = \frac{H_{total}}{number\ of\ stages}$$

Fan Affinity Laws

The Fan Affinity Laws are a set of mathematical relationships that allow engineers to predict how performance parameters (capacity, head, and brake horsepower) change when there are variations in rotational speed, air density, or both. These laws assume that the fan operates at a constant efficiency point on its performance curve.

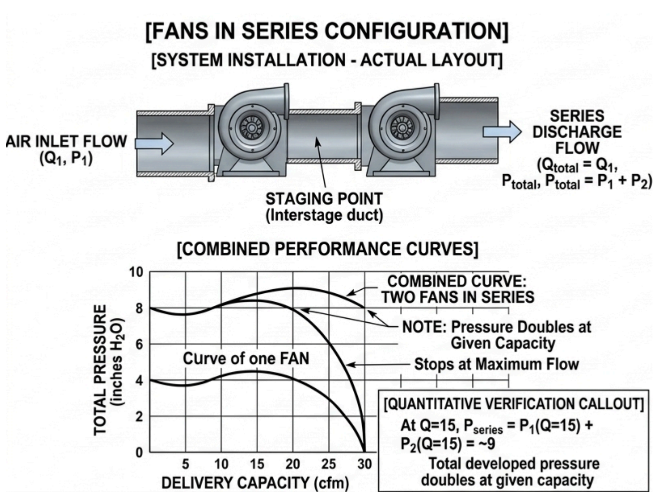
The subscript 1 represents the initial operating conditions, while subscript 2 represents the modified operating conditions.

	Case 1: Variation in Speed $\left(N_1/N_2,\ \rho_1 = \rho_2\right)$	Case 2: Variation in Density $\left(N_1 = N_2,\ \rho_1 \neq \rho_2\right)$	Case 3: Variation in Speed and Density $\left(N_1 \neq N_2,\ \rho_1 \neq \rho_2\right)$
Capacity (Q)	$\frac{Q_1}{Q_2} = \frac{N_1}{N}$	$Q_1 = Q_2$	$\frac{Q_1}{Q_2} = \frac{N_1}{N}$
Head (h)	$\frac{h_1}{h_2} = \left(\frac{N_1}{N_2}\right)^2$	$\frac{h_1}{h_2} = \frac{\rho_1}{\rho_2}$	$\frac{h_1}{h_2} = \frac{\rho_1}{\rho_2}\left(\frac{N_1}{N_2}\right)^2$
Brake Power (BHP)	$\frac{BHP_1}{BHP_2} = \left(\frac{N_1}{N_2}\right)^3$	$\frac{BHP_1}{BHP_2} = \frac{\rho_1}{\rho_2}$	$\frac{BHP_1}{BHP_2} = \frac{\rho_1}{\rho_2}\left(\frac{N_1}{N_2}\right)^3$

In industrial ventilation, mining shaft operations, and large HVAC systems, a single fan may not be sufficient to satisfy both the required volumetric flow rate and the system pressure resistance. Under these conditions, mechanical engineers design multi-fan arrangements. Understanding how fans behave in **series** or **parallel** configurations—and how their performance curves combine—is a staple of fluid machinery board exams.

Fans in Series (Staged Configuration)

Operating fans in series involves staging multiple fans in a sequential, inline arrangement where the discharge duct of the preceding fan connects directly to the suction inlet of the subsequent fan.



A. Operational Characteristics

When fans are installed in series, the same mass flow of air passes through each unit consecutively. Therefore, at any given volumetric capacity, the total pressure developed by the combination is the sum of the pressures produced by each individual fan.

- **Flow Capacity (Q):** Remains constant through all stages.

$$Q = Q_1 = Q_2 = \ldots = Q_n$$

- **Total Developed Pressure (P_{total} or H_T):** Sum of the pressure heads developed by the individual fans.

$$\mathbf{P_{total}} = P_1 + P_2 + \ldots + P_n$$

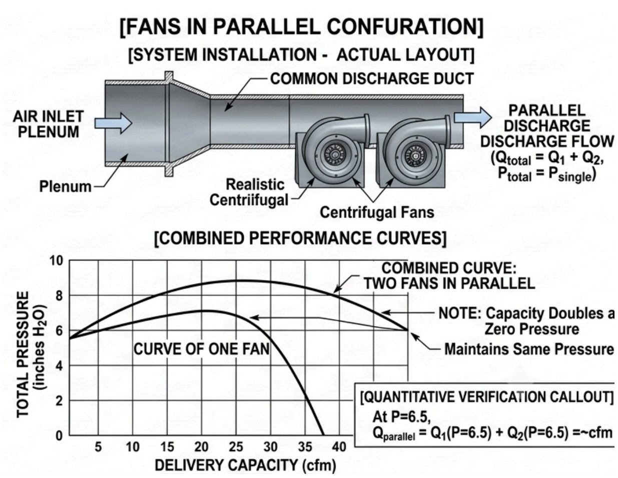
B. Analyzing the Series Performance Curve

When plotting the performance of two identical fans operating in series against the curve of a single fan, we observe the following:

- At **zero delivery capacity** (shut-off head), the total pressure generated by the series setup is exactly **double** that of a single fan (2 x P_{single}).
- At the **maximum free delivery capacity** (zero pressure), the flow rate does not increase; it terminates at the same maximum flow boundary as a single fan (in the provided diagram, this limits at 30 units of capacity).
- The overall curve is stretched vertically, making series operations ideal for overcoming high system resistance (such as long duct runs, extensive filters, or high-pressure dampers).

Fans in Parallel (Side-by-Side Configuration)

Operating fans in parallel involves installing multiple fans side-by-side, where all units draw air from a common inlet plenum and discharge into a shared outlet duct.



A. Operational Characteristics

When fans run in parallel, the total pressure head across all branches is identical, while the total volume flow rate is split among the operating units. If identical fans are used, the total system capacity is the sum of the capacities of the individual fans while maintaining the same operating pressure.

- **Total Developed Pressure (P_{total} or H_T):** Remains equal to the pressure developed by a single branch.

$$\mathbf{P_{total}} = P_1 = P_2 = \ldots = P_n$$

- **Flow Capacity (Q):** Sum of the individual volumetric flow rates.

$$Q = Q_1 + Q_2 + \ldots + Q_n$$

B. Analyzing the Parallel Performance Curve

When plotting the performance of two identical fans operating in parallel, the resulting combined curve shows:

- At **zero delivery capacity** (shut-off head), the developed pressure does not double; it remains exactly equal to the shut-off pressure of a single fan.
- At **zero pressure** (free delivery), the flow capacity is exactly **doubled** (in the provided diagram, the maximum flow limit shifts from 25 units to 50 units).
- The overall curve is stretched horizontally, making parallel configurations highly effective for systems that require high volumetric flow rates under relatively low static system resistance.

If non-identical fans are combined in parallel, engineers must be careful to avoid a phenomenon called **backflow** (or fan stall). If one fan generates a much higher shut-off pressure than the other, it can force air backward through the weaker fan, resulting in system instability and motor damage.

INDUSTRIAL COMPRESSORS

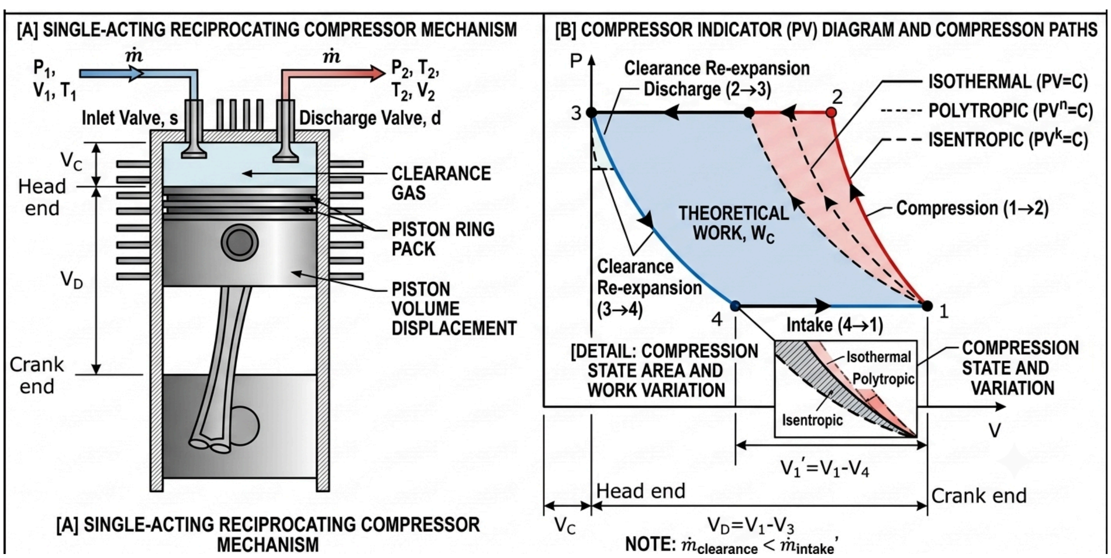
Compressors are fluid machines designed to transfer gases from one location to another by raising the fluid to a higher differential pressure range. Unlike fans and blowers which treat gases as virtually incompressible media, compressors operate across high-pressure differentials where significant gas volume reduction and thermodynamic changes occur. These machines are broadly classified into two major categories: **dynamic (continuous-flow or turbo-compressors)**, which use high-speed rotating impellers to impart kinetic energy that converts to pressure, and **positive displacement (intermittent-flow compressors)**, which physically trap a specific volume of gas and force it into a smaller space to increase its pressure.

Reciprocating Compressor

Reciprocating compressors belong to the positive displacement, intermittent-flow family. They typically operate from a baseline pressure of 35 psig up to standard single-stage limits of 250 psig, and are highly valued in industrial applications due to their high overall efficiency, wide range of capacity configurations, operational simplicity, and compactness. The cylinder design determines whether the system is single-acting or double-acting:

- **Single-acting Reciprocating Compressor:** Compression occurs on only one side of the piston (the head end) per revolution of the crankshaft. The other side (the crank end) is open to the crankcase or does not perform pressure work.
- **Double-acting Reciprocating Compressor:** Compression takes place on both sides of the piston (both head end and crank end). For the same motor rotation speed, double-acting units deliver exactly twice the volumetric capacity of a single-acting compressor. Because compressing gas continuously on both sides generates substantial thermal energy, these units are typically water-cooled and selected for high-capacity heavy industrial systems.

To evaluate the physical velocity and sweeping metrics of the compressor, engineers track Piston Speed (v) and Volume Displacement (V_D).



1. Piston Speed (v)

Piston speed measures the total linear distance traveled by the piston inside the cylinder per unit time. Because the piston travels the length of the stroke (L) twice for every complete crankshaft revolution, the velocity equation is formulated as:

$$v = \frac{2LN}{60}$$

Where:

- v = Mean piston speed, measured in meters per second (m/s).
- L = Length of the piston stroke, measured in meters (m).
- N = Compressor rotational angular speed, measured in revolutions per minute (rpm).

2. Volume Displacement (V_D)

Volume displacement represents the total geometric volume swept by the piston per unit time. Combining the cross-sectional bore area with the stroke details yields the standard engineering calculation:

$$V_D = \left(\frac{\pi D^4}{4}\right)\left(\frac{LN}{60}\right)(no.\ of\ cylinders)(no.\ of\ piston\ actions)$$

Where:

- D = Piston diameter (bore), measured in meters (m).
- L = Piston stroke length, measured in meters (m).
- N = Compressor angular speed, measured in rpm
- No. of piston actions = Set to **1** for single-acting compressors, and **2** for double-acting compressors.

Indicator Diagrams and Capacity Metrics

A typical reciprocating compressor cannot exhaust its entire volume during the discharge stroke. A small gap must remain between the piston head and the cylinder top at the end of the stroke to prevent mechanical impact. This gap is known as the clearance volume (V_C).

1. Compressor Capacity ($V_{1'}$)

Compressor capacity—also referred to as the free air delivery or actual intake volume—is the true net volume of air drawn into the cylinder by the compressor, measured precisely under the ambient intake temperature and pressure conditions (P_1, T_1).

As shown on the P-V diagram, when the high-pressure gas trapped in the clearance volume (V_c or V_3) expands down to the suction pressure along path 3 → 4, it occupies a volume V_4. The suction valve only opens once the pressure inside the cylinder drops below the line pressure, restricting the fresh intake to the remaining stroke volume:

$$V_{1'} = V_1 - V_4$$

2. Thermodynamic Ideal Gas Relation

The volume rate of fresh gas drawn into the system boundary can also be cross-examined using the ideal gas equation of state at inlet conditions:

$$V_{1'} = \frac{m_a R T_1}{P_1}$$

Where:

- m_a = Mass flow rate of air or gas (kg/s).
- R = Ideal gas constant. For standard air calculations, R_{air} = 0.287 kJ/kg-K.
- T_1 = Absolute intake temperature, measured in Kelvin (K).
- P_1 = Absolute intake pressure, measured in kilopascals (kPa).

III. Compressor Work Calculations (W_C)

The work required to drive a single-stage reciprocating compressor depends on the thermodynamic path the gas follows during compression. On the P-V diagram, this work corresponds to the enclosed area 1-2-3-4. All pressures (P_1, P_2) must be evaluated using absolute values.

(a) Work for Polytropic Compression (1 < n < 1.4)

In most practical compressors, heat transfer occurs through the cylinder walls, causing the process to follow a polytropic path defined by $PV^n = C$:

$$W_C = \frac{nP_1V_{1'}}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

(b) Work for Isentropic Compression (n = k = 1.4)

If the cylinder is perfectly insulated and no heat is transferred to the surroundings, the process follows an adiabatic, reversible (isentropic) path defined by $PV^k = C$:

$$W_C = \frac{kP_1V_{1'}}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1\right]$$

(c) Work for Isothermal Compression (n = 1)

If ideal cooling occurs during compression, keeping the gas temperature perfectly constant $\left(T_1 = T_2\right)$, the process follows an isothermal line (PV = C). This pathway requires the least amount of work, making it the thermodynamic ideal target for compressor efficiency:

$$W_C = P_1V_{1'}\, ln\left(\frac{P_2}{P_1}\right)$$

Pressure-Volume-Temperature (PVT) Relationships

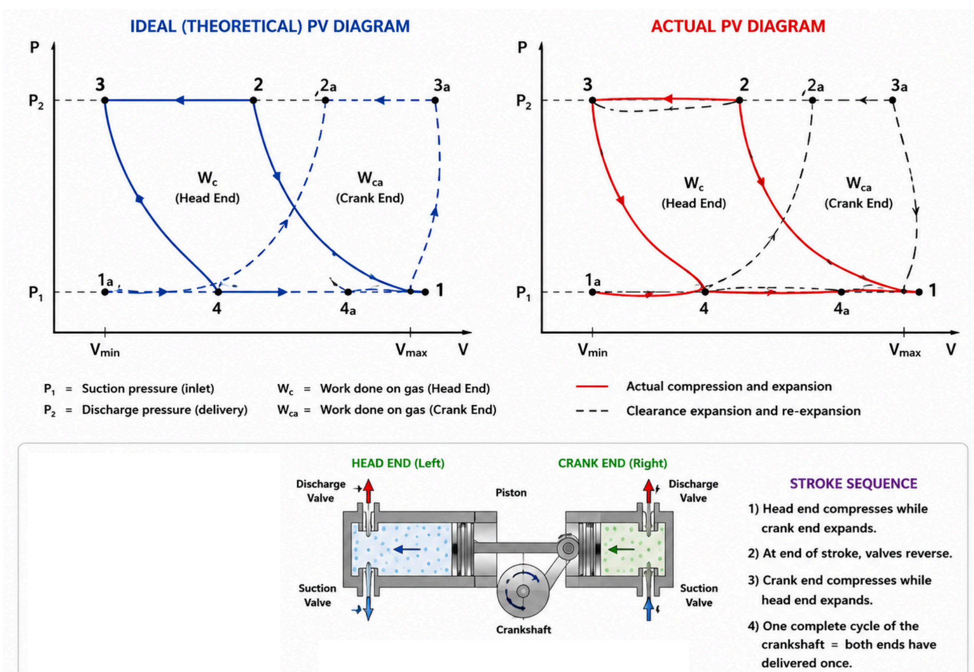
When solving multi-stage or single-stage compressor problems on the board exam, you must often find missing temperature or volume properties at the discharge state. For non-isothermal processes, apply the standard ideal gas relation:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

- **Polytropic exponent (n):** Falls between 1 < n < 1.4.
- **Isentropic exponent (k):** Set to n = k = 1.4 for standard air.
- **Isothermal state:** Set to n = 1, meaning $\left(T_1 = T_2\right)$.

Double-Acting Reciprocating Compressors

A double-acting reciprocating compressor represents a higher level of mechanical efficiency compared to single-acting configurations. While a single-acting machine utilizes only one side of the piston to compress gas, a double-acting compressor utilizes **both sides of the piston**—the head end and the crank end—to compress gas alternatively during every single stroke of the crankshaft.



Operational Characteristics

- **Volumetric Multiplier:** For the exact same motor rotation speed (N), a double-acting compressor delivers **twice the volumetric capacity** of a single-acting counterpart. This occurs because as one side of the piston undergoes its compression/discharge stroke, the opposing side simultaneously executes its suction/intake stroke.

- **Thermal Management:** Compressing gas continuously on both sides of a single piston assembly generates a significant amount of heat. Because of this high thermal loading, double-acting systems are fundamentally **water-cooled** and are primarily deployed in heavy-duty, large-capacity industrial applications.

II. Double-Acting Indicator Diagrams and Work Profiles

The indicator diagram (P-V plot) for a double-acting compressor maps out two distinct loops that occur simultaneously within a single crank cycle. The total work required by the machine (W_{total}) is the sum of the work performed on both sides of the cylinder.

COMPRESSOR EFFICIENCIES AND PERFORMANCE METRICS

This module isolates the core performance evaluation indexes of reciprocating compressor systems: **Volumetric Efficiency (η_V)** and **Compressor Efficiency (η_C)**. Understanding the geometric constraints, the mathematical derivation of clearance variables, and energy conversion thresholds is vital for single-stage and multi-stage compressor board exam problems.

I. Volumetric Efficiency (η_V)

Volumetric efficiency describes how efficiently air or gas is drawn into the cylinder of the air compressor. It serves as a structural metric comparing the actual intake capability of the machine against its theoretical geometric swept bounds. Specifically, it is defined as the ratio of the actual volume of air drawn in divided by the volume displacement (or the maximum possible amount of air that can be drawn in under ideal conditions):

$$\eta_V = \frac{actual\ volume}{volume\ displacement}\ x\ 100\% = \frac{V_{1'}}{V_D}\ x\ 100\%$$

As detailed in previous modules, the fresh air taken in per cycle is restricted because the high-pressure gas trapped within the clearance volume must first expand down to the suction pressure line before the inlet valves can actuate. Therefore, we substitute $V_{1'} = V_1 - V_4$ to yield:

$$\eta_V = \frac{V_1 - V_4}{V_D}\ x\ 100\%$$

A. Clearance Ratio (c)

To express this relationship purely in terms of operational pressures and the cylinder design geometry, we define the **percent clearance (c)**, which represents the ratio of the clearance volume (V_3 or V_C) to the volume displacement (V_D). In industrial applications, c typically ranges from 3% to 10% ($0.03 \le c \le 0.10$):

$$c = \frac{V_3}{V_D} \qquad \Rightarrow \qquad V_3 = cV_D$$

The total maximum volume within the cylinder chamber (V_1) occurs when the piston is fully retracted to the crank end. This absolute volume is the sum of the clearance volume and the swept displacement volume:

$$V_1 = V_3 + V_D = cV_D + V_D$$

During the re-expansion stroke (from state 3 → 4), the trapped clearance gas follows a polytropic path. Applying standard pressure-volume-temperature thermodynamic relationships across these bounds gives:

$$\left(\frac{V_4}{V_3}\right)^n = \left(\frac{P_3}{P_4}\right) \qquad \Rightarrow \qquad \left(\frac{V_4}{V_3}\right) = \left(\frac{P_3}{P_4}\right)^{\frac{1}{n}}$$

Given that state 3 sits on the discharge line ($P_3 = P_2$) and state 4 rests on the suction line $\left(P_4 = P_1\right)$, we isolate the expanded volume V_4 as follows:

$$V_4 = V_3\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} = cV_D\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

B. The Standardized Volumetric Efficiency Equation

Substituting these definitions of V_1 and V_4 back into the primary volumetric equation highlights how displacement cancels out as a factor:

$$\eta_V = \frac{\left(cV_D + V_D\right) - cV_D\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}}{V_D} \, x\,100\%$$

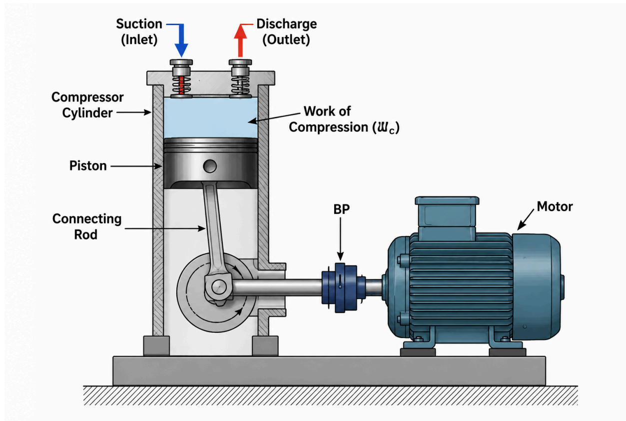
Factoring out V_D from the numerator and denominator yields the final standard engineering formula for volumetric efficiency:

$$\eta_V = \left[1 + c - c\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right] x\,100\%$$

The pressure ratio term $\frac{P_2}{P_1}$ must always be calculated using absolute pressures. This equation shows that as the discharge pressure (P_2) increases, or as the physical clearance ratio (c) grows, the volumetric efficiency drops, which reduces the actual mass flow capacity of the compressor.

II. Compressor Efficiency (η_C)

While volumetric efficiency measures structural capacity, **compressor efficiency (η_C)** evaluates energy conversion performance.



As illustrated in the system diagram, a prime mover (such as an electric motor or internal combustion engine) supplies a gross mechanical input power (BP) to the shaft assembly. Mechanical losses—such as friction in the bearings, connecting rod journals, and piston ring interfaces—mean that only a portion of this energy is converted into fluid work (W_C) to compress the gas.

It is defined as the ratio of the fluid power output (the theoretical work transmitted directly to the gas inside the chamber, W_c) to the mechanical brake power input delivered to the compressor shaft (BP):

$$\eta_C = \frac{W_C}{BP} \, x \, 100\%$$

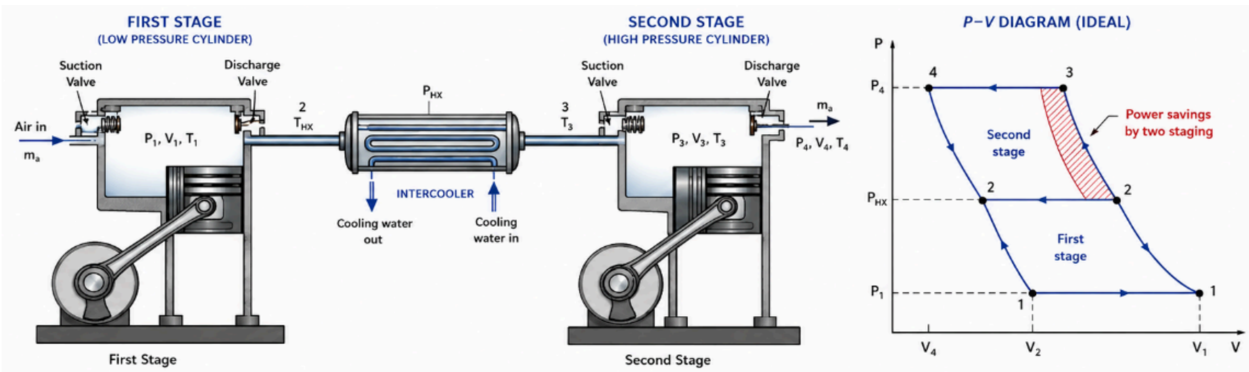
Multi-stage Reciprocating Compressors

In single-stage compression systems, attempting to reach very high discharge pressures significantly increases the final discharge temperature and severely reduces the volumetric efficiency due to the re-expansion of clearance gas. To overcome these limitations, industrial applications utilize multi-stage reciprocating compressors.

In a multi-stage compressor setup, the overall pressure rise is broken down across two or more separate cylinders working in series. The gas is first drawn into a low-pressure cylinder (First Stage), where it undergoes a preliminary pressure increase. It is then discharged into an intermediate chamber or heat exchanger, known as an **intercooler**, before entering the high-pressure cylinder (Second Stage) for final compression to the ultimate system discharge pressure (P_4).

Two-Stage Compressor

A critical concept for the board exam is determining what dictates the overall capacity of a multi-stage machine. In a multi-stage arrangement, the low-pressure cylinder (First Stage) uniquely determines the volumetric efficiency and the total volume capacity of the entire compressor system.



This occurs because the first-stage cylinder is the only component that interfaces directly with the ambient fresh air intake source (P_1, T_1). Because the cylinders operate in series within a closed system loop, whatever mass flow rate (m_a) or volumetric capacity ($V_{1'}$) the low-pressure cylinder successfully captures, processes, and passes to the succeeding stages must eventually be discharged by the final stage. If the first stage cannot draw the gas in, the subsequent high-pressure stages have no mass to compress.

Intercooling Mechanics and Power Savings

The primary thermodynamic advantage of multi-staging is the reduction of total compression work through intercooling. As gas is compressed in the first stage, its temperature rises rapidly from T_1 to an elevated intercooler inlet temperature (T_{HX} or T_2).

The Intercooling Process

The hot gas enters the intercooler at the intermediate pressure (P_{HX}). Inside this heat exchanger, heat is rejected to an external cooling medium (typically water or ambient air), dropping the gas temperature from T_{HX} down to a lower second-stage inlet temperature (T_3). In an ideal, perfectly *intercooled* system, the gas is cooled back down to its initial ambient suction temperature, meaning: $T_3 = T_1$

Analysis of the P-V Diagram & Power Savings

When looking at the Pressure-Volume (**P-V**) indicator diagram, single-stage compression to a high pressure causes the polytropic compression line to curve steeply to the right, tracking towards a high final temperature.

By splitting the process into two stages with an intermediate cooling step, the following changes occur on the diagram:

1. **First Stage:** The gas is compressed from the initial suction pressure (P_1) up to the intercooler pressure (P_{HX}).
2. **Intercooling Step:** Cooling the gas at a constant intermediate pressure (P_{HX}) causes its volume to shrink. On the diagram, this shows up as a horizontal shift to the left, moving from the end of the first stage to the start of the second stage.
3. **Second Stage:** Compression begins from a smaller volume at P_{HX} and continues up to the final discharge pressure (P_4).

This horizontal shift to a smaller volume before starting the second stage creates a distinct, enclosed area on the P-V diagram between the single-stage path and the multi-stage path. This shaded area represents the **power savings achieved by two-staging**. Because the second cylinder compresses a denser, cooler gas, it requires less total mechanical energy input than a single-stage machine would to achieve the same pressure rise.

By implementing a multi-stage compressor setup, the required mechanical power input to the system is significantly lessened. Simultaneously, the gas discharge temperature across individual stages and the internal pressure differentials are minimized, preventing thermal stress on cylinder components and lubricating oils.

A. Intermediate Pressure Determination

For a two-stage compressor, the optimal intermediate intercooler pressure (P_{HX}) that balances the load evenly and minimizes total work input can be theoretically approximated using a geometric mean:

$$P_{HX} = \sqrt{P_1 P_4}$$

Where:

- P_1 = Absolute pressure intake at the first stage.
- P_4 = Absolute pressure at the second stage discharge.

B. Two-Stage Compressor Work ($W_{C1\text{-}2}$)

When a two-stage compressor operates under ideal conditions (with perfect intercooling and equal pressure ratios across stages), the total work required by the two-stage machine (**$W_{C1\text{-}2}$**) is exactly double the work of the first stage:

$$\mathbf{W_{C1\text{-}2}} = \frac{2nP_1V_{1'}}{n-1}\left[\left(\frac{P_{HX}}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

Where $V_{1'}$ is the actual compressor capacity measured at the first-stage inlet, and n represents the polytropic exponent index.

C. Intercooler Thermal Load (Q_{HX})

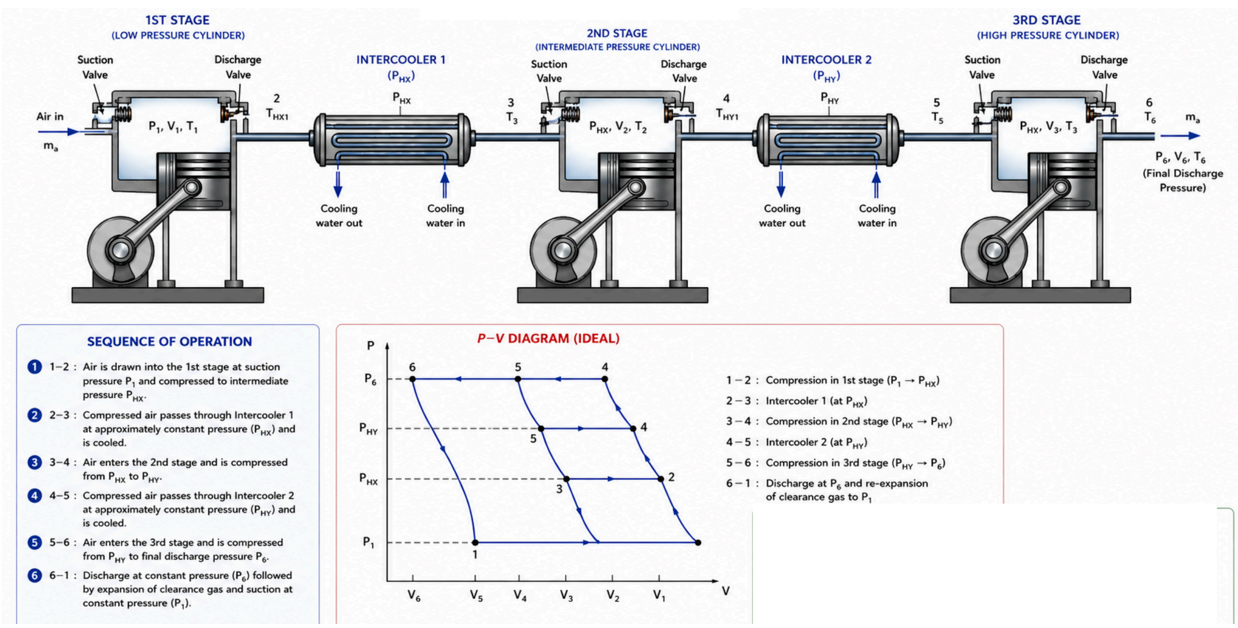
The heat rejected by the intercooler exchanger (**Q_{HX}**) to cool the gas back down to the initial intake temperature is calculated via energy balance:

$$Q_{HX} = m_a C_p \left(T_1 - T_{HX}\right)$$

Where m_a is the mass flow rate of the air, C_p is the specific heat capacity at constant pressure, T_{HX} is the hot gas temperature leaving the first stage, and T_1 is the target cooled intake temperature.

Three-Stage Compressor Configurations

For higher pressure applications, a three-stage reciprocating configuration is utilized, where the gas undergoes three successive compression processes interleaved by two distinct intercooling stages.



As depicted in the three-stage schematic and corresponding P-V indicator diagram:

1. Gas enters the 1st stage at suction pressure P_1 and is compressed to the first intermediate intercooler pressure P_{HX}.
2. After passing through the first intercooler, it enters the 2nd stage and is compressed from P_{HX} to a second, higher intermediate pressure P_{HY}.
3. Following a second intercooling phase, the 3rd stage compresses the gas from P_{HY} to its ultimate system terminal discharge pressure P_6.

A. Intermediate Pressures (P_{HX} and P_{HY})

Under ideal design parameters, the intermediate pressures P_{HX} and P_{HY} can be theoretically approximated using the terminal pressures (P_1 and P_6):

$$P_{HX} = \sqrt[3]{P_1^{\ 2} P_6}$$

$$P_{HY} = \sqrt[3]{P_1 P_6^{\ 2}}$$

Where:

- P_1 = Absolute pressure intake at the first stage.
- P_6 = Absolute pressure at the third stage discharge.

B. Three-Stage Compressor Work ($W_{C,1-2-3}$)

Assuming identical pressure ratios and perfect intercooling across all three distinct chambers, the total required power input is exactly three times the work done in a single stage:

$$W_{C,1-2-3} = \frac{3nP_1V_{1'}}{n-1}\left[\left(\frac{P_{HX}}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

III. General Formula for Multi-Stage Compressors ("m" Number of Stages)

When expanding multi-stage theory to any arbitrary "m" number of stages, generalized mathematical equations are employed to solve for total cycle work and optimization boundaries.

A. Ideal Design Conditions

For a perfectly optimized system with equal work distribution and optimal efficiency, the pressure ratios across every single individual stage must be perfectly identical:

$$\frac{P_{HX}}{P_1} = \frac{P_{HY}}{P_X} = \ . \ . \ . \ = \frac{P_F}{P_{prev}}$$

For instance, applying this equality rule to a three-stage machine yields:

$$\frac{P_{HX}}{P_1} = \frac{P_{HY}}{P_X} = \frac{P_6}{P_{HY}}$$

B. Generalized Pressure Ratio & First Intermediate Pressure (P_{HX})

For any m-stage machine, the first intermediate pressure stage boundary (P_{HX}) can be calculated using the general relationship:

$$P_{HX} = \sqrt[m]{\left(P_1\right)^{m-1} P_F}$$

Where:

- P_1= Primary suction inlet pressure.
- P_F = Final system discharge pressure.
- m = Total number of compression stage

C. Generalized Total Compressor Work (W_{Cm})

The absolute work required to operate an optimized m-stage compressor (**W_{Cm}**) is formulated by applying the multiplier m directly to the primary stage energy calculus:

$$W_{C,m} = \frac{m \cdot n P_1 V_{1'}}{n-1}\left[\left(\frac{P_{HX}}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$