In hydroelectric power systems, a **penstock** is the high-pressure pipeline that carries water from a reservoir or head gate down to the turbine nozzle. During this journey, some of the water's initial potential energy is converted into thermal energy (heat) due to boundary friction against the pipe walls. This is called **friction head loss (h_L or h_f)**.

While Darcy's Equation is the standard formulation globally, some local board examinations utilize the **Morse Equation** for head loss:

Darcy Equation: $h_f = \frac{f \cdot L \cdot v^2}{2 \cdot g_o \cdot D}$ vs. Morse Equation: $h_f = \frac{2 f \cdot L \cdot v^2}{g_o \cdot D}$

Where:

- h_f = h_L =$ friction head loss (m)
- f = coefficient of friction (dimensionless)
- L = total length of the penstock (m)
- v = fluid velocity ({m/s)
- g_o = acceleration due to gravity (9.81m/s^2)
- D = inside penstock diameter (m)

Torricelli's Theorem and Effective Head

The total vertical fall from the head water level to the centerline of the nozzle is the **gross head (H_g)**. The net pressure head driving the fluid through the nozzle is the **effective head (H_e)**. Subtracting the losses from the gross head yields:

$$H_e = H_g - h_f$$

Applying **Torricelli's Theorem** (derived from Bernoulli's equation), the potential energy of this net water column converts entirely to kinetic energy at the nozzle tip, giving us the exit velocity (v):

$$v = \sqrt{2 g_o H_e}$$

<u>Step 1:</u> Calculate Friction Head Loss (h_f) & Effective Head (H_e)

The problem specifies that the friction head loss is 10% of the elevation drop (H_g = 30 m):

$$h_f = 3.0 \text{ m}$$

Now, compute the net effective head available to drive the Pelton jet:

$$H_e = (30 - 3)\text{m} = 27\text{m}$$

Step 2: Compute Jet Velocity (v)

Using Torricelli's equation, determine the flow velocity exiting the nozzle:

$$v = \sqrt{2g_o H_e} = \sqrt{2\left(9.81\ \frac{m}{s^2}\right)(27\ m)} = 23.016\ \frac{m}{s}$$

Step 3: Solve for Penstock Diameter (D)

Using the **Morse Equation** as defined by the textbook criteria:

$$h_f = \frac{2\,f \cdot L \cdot v^2}{g_o \cdot D}$$

$$3.0m = \frac{2\,(0.00093)(85m)\left(23.016\ \frac{m}{s}\right)^2}{\left(9.81\ \frac{m}{s^2}\right)D}$$

D = 2.846 m

Step 4: Calculate Volume Flow Rate (Q) and Water Power (WP)

The volume flow rate (Q) passing through the penstock is determined by the cross-sectional area (A) and velocity (v):

$$Q = Av = \frac{\pi D^2}{4} v = \frac{\pi (2.846\ m)^2}{4}\left(23.016\ \frac{m}{s}\right)$$

$$Q = 146.40\ \frac{m^3}{s}$$

Now, compute the net hydraulic power (Water Power, WP) transmitted by the fluid jet using a specific weight of water (γ_W = 9.81 kN/m^3):

$$WP = Q\gamma_W H_e = \left(146.40\ \frac{m^3}{s}\right)\left(9.81\ \frac{kN}{m^3}\right)(27\ m)$$

WP = 38,776 kW

This problem illustrates a key engineering tradeoff in fluid power systems. While a larger penstock diameter reduces flow velocity and minimizes frictional head losses, it dramatically increases material weight, construction costs, and structural loading requirements.

As an engineer reviewing for the board exam, the key takeaway is to **always match the equation to the given coefficient's convention**:

- If the problem gives a standard "Darcy friction factor" or simply "friction factor (f)" without qualifiers, default to the **Darcy-Weisbach equation** containing the $\frac{1}{2g_o}$ term.
- If the problem explicitly specifies **"Morse"** or **"Fanning"**, use the **Morse equation** containing the $\frac{2}{g_o}$ term to ensure your mathematical models align perfectly with the examiner's answer key.