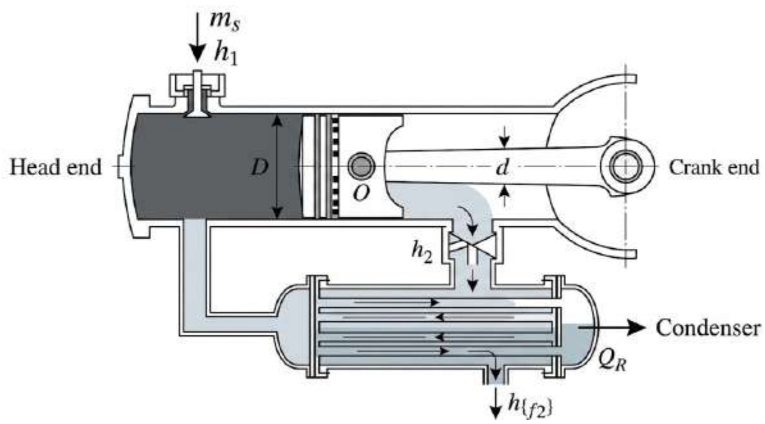
STEAM ENGINE

A **steam engine** is an external combustion, double-acting reciprocating machine that utilizes high-pressure steam as a working fluid to convert thermal energy into mechanical work. Unlike typical internal combustion engines that operate on single-acting cycles (where power is delivered to only one side of the piston), a classic industrial steam engine acts on both the head end and the crank end of the cylinder during a single crankshaft revolution. High-pressure steam enters through an admission valve to press against one side of the piston while the opposite side exhausts low-pressure, expanded steam toward a condenser. As the piston reaches the end of its linear stroke, the valve mechanism reverses the fluid routing. Steam is then admitted to the opposite side, driving the piston back in the reverse direction. This double-acting design effectively doubles the power strokes per revolution compared to a single-acting system operating at identical rotational speeds.



As steam flows through the engine, it undergoes sequential thermodynamic processes: admission at a high enthalpy state (h_1), expansion within the cylinder volume to execute work, and exhaust at a lower enthalpy state (h_2). The spent steam then travels to a condenser, where it rejects heat (Q_R) to a cooling medium, changing phase into a liquid condensate at a specific saturated liquid enthalpy (h_{f2}). This continuous cycling relies heavily on accurate mechanical geometry to calculate structural limits, volumetric capacities, and power capabilities.

Displacement Volume

The volume displacement (V_D) represents the total space swept by the engine's reciprocating piston per unit of time. Because industrial steam engines are inherently double-acting, the geometric calculation must account for the mechanical work occurring on both faces of the piston.

Case A: Neglecting the Piston Rod Cross-Section

When the cross-sectional area of the piston rod is small enough to be considered negligible, both the head end and the crank end are assumed to possess an identical effective surface area equal to the full circular area of the cylinder bore (D). Under this assumption, the total volume displacement per second is formulated by doubling the volumetric displacement of a single-acting cylinder:

$$V_D = 2\left[\frac{\pi}{4}D^2L\left(\frac{N}{60}\right)\right]$$

Where:

- V_D = Total volumetric displacement (m^3)
- D = Bore diameter of the cylinder (m)
- L = Piston stroke length (m)
- N = Engine rotational speed in revolutions per minute (rpm), making $\frac{N}{60}$ the revolutions per second (rps).

Case B: Accounting for the Piston Rod Cross-Section

In precise engineering applications, the piston rod extending through the crank end of the cylinder cannot be ignored. The rod occupies a physical volume, thereby reducing the net surface area available for the steam to exert pressure on during the crank-end power stroke. The effective area of the head end remains $\frac{\pi}{4}D^2$, while the net effective area of the crank end is reduced to $\frac{\pi}{4}\left(D^2 - d^2\right)$, where d is the diameter of the piston rod. Combining both active faces, the mathematically exact expression for displacement volume becomes:

$$V_D = \frac{\pi}{4}\left[D^2 + \left(D^2 - d^2\right)\right]L\left(\frac{N}{60}\right)$$

Indicated Power and Indicator Diagrams

Indicated Power (IP) represents the total theoretical power developed within the engine cylinder by the expansion of the working fluid, ignoring any subsequent mechanical or friction losses. It is calculated directly from the mean effective pressure inside the cylinder chamber:

$$IP = P_{mi}V_D$$

Where P_{mi} is the indicated mean effective pressure (N/m^2 or Pa) and V_D is the displacement volume (m^3/s).

To determine the true performance of an operating engine, an diagnostic tool known as an engine indicator is mechanically linked to the cylinder. This device traces a closed pressure-volume (P-V) diagram representing the actual cycle. The indicated mean effective pressure is then extracted by analyzing the physical area of this plotted loop.

Evaluation via Average Indicator Card Area

If the engine indicator generates a continuous trace loop with an easily measurable total area, the average indicated mean effective pressure is given by:

$$P_{mi} = \frac{A_c S_c}{L_c}$$

Where:

- A_C = Measured area of the indicator diagram card (mm^2 or cm^2)
- L_C = Physical length of the indicator diagram card (mm or cm)
- S_C = Spring scale or spring constant of the indicator mechanism (kPa/mm or bar/cm), representing the scale factor converting physical card height into actual fluid pressure.

Evaluation via Distinct Head-End and Crank-End Profiles

Because flow and valve timing can vary slightly between the two sides of a double-acting cylinder, indicators often capture distinct traces for each end. When the individual card areas for the crank end (A_C) and the head end (A_H) are measured separately over a shared card length (L_C), the representative mean effective pressure is calculated using the arithmetic mean of the two areas:

$$P_{mi} = \frac{\left(\frac{A_C + A_H}{2}\right) S_C}{L_C}$$

While Indicated Power quantifies the internal energy captured from the steam, not all of this power is successfully delivered to the output shaft. A fraction is inevitably lost overcoming mechanical resistance, sliding friction, and rotational windage within the bearings, packing glands, and crosshead guides.

Brake Power (BP)

Brake Power denotes the actual, usable mechanical power delivered directly to the engine's output shaft or flywheel. It can be measured using an external brake dynamometer and is defined using the brake mean effective pressure (P_{mB}):

$$BP = P_{mB} V_D$$

Friction Power (FP)

Friction Power represents the rate of energy dissipation due to internal mechanical resistance. It serves as the mathematical bridge between the theoretical energy produced inside the cylinder and the actual energy output at the shaft:

$$FP = IP - BP$$

Engineers use these relationships to troubleshoot internal engine performance. A drop in mechanical efficiency points to lubrication issues or worn mechanical components, whereas a drop in indicated power points to thermal energy losses, such as leaking valves or worn piston rings.

STEAM ENGINE EFFICIENCIES

In steam plant engineering, evaluating performance requires distinguishing between mechanical, thermal, and engine efficiencies. Each efficiency term measures a distinct boundaries of energy conversion.

Mechanical Efficiency (η_M)

The mechanical efficiency of the steam engine measures its mechanical transmission performance. It evaluates the percentage of indicated power that survives friction losses to become usable brake power:

$$\eta_M = \frac{BP}{IP} * 100\% = \frac{P_{mB}}{P_{mi}} * 100\%$$

Thermal Efficiencies

Thermal efficiencies assess the engine's ability to convert raw thermal energy input into mechanical work. The baseline heat input rate supplied to the steam generator loop to produce the steam is defined as $Q_{in} = m_s(h_1 - h_{f2})$, where m_s is the steam mass flow rate, h_1 is the superheated or saturated steam enthalpy entering the engine, and h_{f2} is the enthalpy of saturated liquid condensate returning from the condenser.

Indicated Thermal Efficiency (η_{IPth})

Indicated thermal efficiency represents the ratio of the power generated inside the cylinder (Indicated Power) to the total rate of heat energy supplied to the steam loop:

$$\eta_{IPth} = \frac{IP}{m_s(h_1 - h_{f2})} * 100\%$$

Brake Thermal Efficiency (η_{BPth})

Brake thermal efficiency represents the net system efficiency. It calculates the ratio of the final usable shaft power (Brake Power) to the total rate of heat energy supplied to the steam loop:

$$\eta_{BPth} = \frac{BP}{m_s(h_1 - h_{f2})} * 100\%$$

Engine Efficiencies

Engine efficiency (often referred to as *isentropic engine efficiency* or *relative efficiency*) isolates the performance of the engine cylinder itself. Instead of using the overall loop heat input in the denominator, it compares the engine's actual power output to the ideal, theoretical work available during an isentropic expansion from the inlet state to the exhaust pressure.

The ideal isentropic enthalpy drop is represented by $(h_1 - h_2)$, where h_2 is the ideal/isentropic enthalpy at the exhaust pressure.

Indicated Engine Efficiency (η_{IPe})

Indicated engine efficiency measures how closely the actual expansion process within the cylinder approaches a perfect, frictionless, adiabatic (isentropic) process:

$$\eta_{IPe} = \frac{IP}{m_S(h_1 - h_2)} * 100\%$$

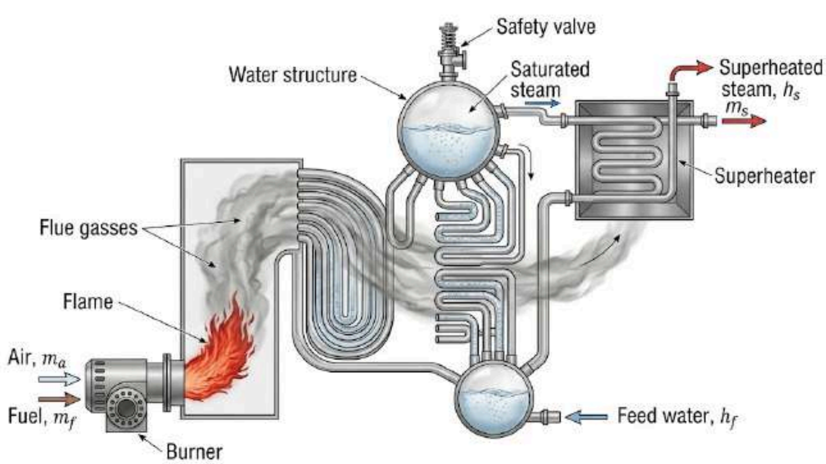
Brake Engine Efficiency (η_{BPe})

Brake engine efficiency relates the final usable shaft power (Brake Power) to the ideal isentropic work rate. It accounts for both fluid expansion losses inside the cylinder and mechanical friction losses along the shaft drive:

$$\eta_{BPe} = \frac{BP}{m_S(h_1 - h_2)} * 100\%$$

BOILERS (STEAM GENERATORS)

A boiler, fundamentally designated in engineering practice as a steam generator, is defined as a closed vessel explicitly engineered to generate steam through the application of heat released during the combustion of fuel. The system acts as a specialized heat exchanger where chemical energy stored within a fuel source (solid, liquid, or gaseous) is transformed via combustion into thermal energy, which is subsequently transferred across metallic heat exchange surfaces to a working fluid—typically liquid water. In the operational cycle, feed water enters the boiler shell or tube network at a specific liquid enthalpy state (h_f). As it circulates past surfaces heated by the passing flue gases, the fluid undergoes a phase transformation to produce saturated steam, which may be routed through a secondary superheater stage to yield high-energy superheated steam at an enhanced enthalpy state (h_s).



Industrial steam generators are divided into two distinct structural classifications based on the physical boundary arrangement of the fluid and the thermal gases: **Fire Tube Boilers** and **Water Tube Boilers**.

1. Fire Tube Boilers

In a fire tube boiler, the hot products of combustion (flue gases) pass directly through the interior channels of a bank of tubes, while the working fluid (water) is contained within the outer surrounding boiler shell.

- **Economic and Operating Profile:** These units feature a low initial capital cost. Because the outer shell holds a vast volume of water, fire tube units inherently possess a larger water capacity.
- **Thermal Inertia and Load Response:** Due to this large fluid volume, these systems exhibit high thermal inertia, making them slower in coming up to operating pressure from a cold start. However, the large stored energy reserve functions as a thermal buffer, enabling the system to meet sudden load changes quickly without experiencing severe pressure drops.
- **Physical Constraints:** Because the external shell is subjected to full internal steam pressure, structural safety constraints limit fire tube operating parameters to relatively modest pressures ranging from 150 to 300 psi and cap total steam production capacities up to approximately 25,000 lb/hr.

2. Water Tube Boilers

In a water tube boiler, the arrangement is inverted: the working fluid circulates inside the interior channels of the tube network, while the hot flame and combustion flue gases pass along the exterior surfaces of the tubes.

- **High-Pressure Capability:** Because high internal pressures are confined within small-diameter tubes rather than a large exterior shell, these structures can withstand ultra-high mechanical stresses, accommodating operating pressures up to 5000 psi+.
- **Capacity and Adaptability:** Water tube configurations are suited for massive industrial applications, capable of achieving steam production rates scaling to 7 million lb/hr+. Furthermore, their layout is highly customizable, meaning that secondary superheater assemblies can be added easily to maximize the thermal efficiency of downstream turbines.
- **Fuel Flexibility:** The external furnace layout renders water tube configurations highly flexible with different fuels, permitting the combustion of solid pulverized coal, heavy oils, biomass, or gases within the same basic installation.

BOILER RATINGS AND QUANTITATIVE METRICS

2.1 Heat Generation and Fuel Calculations

The total rate of heat energy released within the furnace area is governed by the mass consumption rate of the fuel and its corresponding chemical energy density. The absolute thermal energy generated (Q) per unit of time is expressed as:

$$Q = m_F q_F$$

Where:

- Q = Total rate of heat generated by the fuel (kJ/hr or Btu/hr)
- m_F = Mass flow rate of fuel consumed per unit time (kg/hr or lb/hr)
- q_F = Higher heating value (HHV) of the fuel (kJ/kg or Btu/lb)

Rated Boiler Horsepower (RBH)

Historically and within modern licensing frameworks, Rated Boiler Horsepower (RBH) serves as a standardized index to quantify a steam generator's physical capacity to deliver steam based purely on its structural heat transfer geometry. Rather than evaluating real-time operational states, RBH establishes performance baselines by dividing the total physical heating surface area (HS) by a legally defined standard area allocation per horsepower unit.

- **Water Tube Boiler Specification:** For water tube designs, engineering standards allocate 9.9 ft^2 (0.91 m^2) of physical heating surface area to develop exactly one boiler horsepower:

$$RBH = \frac{HS}{0.91}$$

- **Fire Tube Boiler Specification:** For fire tube configurations, a larger allocation of 12ft^2 (1.1 m^2) of physical heating surface area is required to establish one boiler horsepower due to differences in fluid circulation profiles and thermal transfer rates:

$$RBH = \frac{HS}{1.1}$$

Where HS represents the total active heating surface area measured in square meters (m^2).

Developed Boiler Horsepower (DBH)

While RBH is a rigid geometric property, Developed Boiler Horsepower (DBH) is an operational metric that quantifies the actual thermal performance achieved by the steam generator. By definition, one boiler horsepower is equivalent to the thermal energy required to completely vaporize 34.5{ lb of liquid water at a saturated temperature of 212°F into dry saturated steam at 212°F within the span of one hour.

Derivation of the Boiler Horsepower Equivalent

To establish the absolute metric equivalent, we evaluate the latent heat of vaporization of water at 212ºF, which equals 970.3 Btu/lb. The thermal energy required to develop a single boiler horsepower is derived as follows:

$$\left(970.3\ \frac{Btu}{lb}\right)\left(34.5\ \frac{lb/hr}{1\ Bo.hp}\right) = 33,475\ \frac{Btu}{hr-Bo.hp}$$

Converting this British thermal unit value to SI units yields the standard constant:

$$1\ Bo.\ hp = 33,475\ \frac{Btu}{hr} = 35,316\ \frac{kJ}{hr}$$

DBH Formulation

When operating under real-world conditions, a steam generator handles fluid entering at an arbitrary feed water enthalpy (h_f) and discharges steam at an advanced enthalpy (h_s). The total actual rate of thermal energy absorbed by the fluid is calculated as $m_s(h_s - h_f)$. Dividing this net thermal absorption rate by the isentropic metric horsepower baseline yields the expression for Developed Boiler Horsepower:

$$DBH = \frac{m_s\left(h_s - h_f\right)}{35,316}$$

Where:

- m_s = Mass flow rate of steam generated (kg/hr)
- h_s = Enthalpy of the leaving steam (kJ/kg
- h_f = Enthalpy of the entering feed water (kJ/kg)

Percentage Boiler Rating (%R)

The Percentage Boiler Rating (%R) measures the ratio of actual thermal output to the nominal design capacity of the boiler. It indicates whether a boiler is operating below its rated capacity, at its nominal design limit (100%), or in an over-fired state (>100%):

$$\%R = \frac{DBH}{RBH}\ x\ 100\%$$

Factor of Evaporation (FE)

The Factor of Evaporation (FE) is a dimensionless ratio that normalizes actual operational thermal conditions against the standard baseline condition of vaporization at 212ºF (100ºC). It represents the ratio of the net heat added per pound of water under actual operating conditions to the latent heat of vaporization at atmospheric pressure (970.3 Btu/lb or 2257 kJ/kg):

$$FE = \frac{h_s - h_f}{2257}$$

Volumetric and Mass Evaporation Rates

The Boiler Evaporation Rate (BER), often referred to as the fuel-to-steam ratio, is a key indicator of operational economy. It measures the mass of steam generated per unit mass of fuel consumed:

$$\text{BER} = \frac{m_s}{m_f}$$

Evaluating steam generator performance requires distinguishing between gross internal heat transfer efficiency and net system efficiency, which accounts for external parasitic loads.

Gross Boiler Efficiency (η_B)

Gross Boiler Efficiency (η_B) measures the effectiveness of the heat exchange surfaces in transferring the chemical energy released by the fuel into the working fluid. It is calculated as the ratio of the net thermal energy absorbed by the steam to the total heat energy supplied by the fuel:

$$\eta_B = \frac{m_s\left(h_s - h_f\right)}{m_F q_F}\ x\ 100\%$$

Net Boiler Efficiency ($\eta_{B,\,NET}$)

In modern steam power plants, the generation process requires auxiliary equipment, such as soot blowers, forced-draft fans, induced-draft fans, and automated fuel-feeding mechanisms. These auxiliary systems consume energy, creating an internal parasitic load.

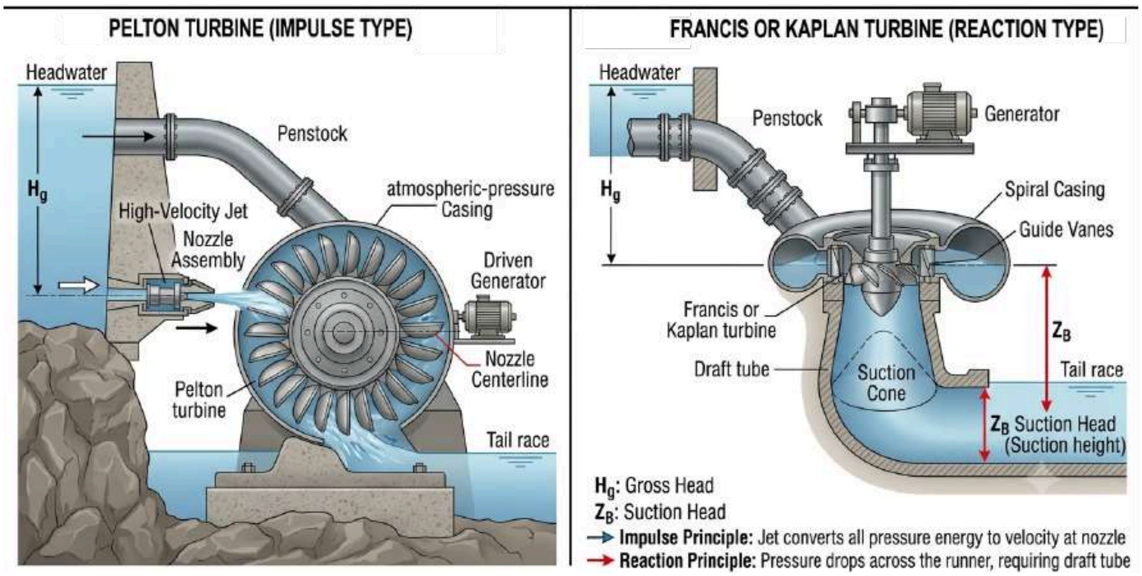
To evaluate the true net thermodynamic contribution of the steam generator to the power plant loop, engineers use the Net Boiler Efficiency ($\eta_{B,\,NET}$). This metric subtracts the thermal equivalent of the auxiliary energy consumption from the gross heat added to the steam before dividing by the total fuel energy input:

$$\eta_{B,\,NET} = \frac{m_s\left(h_s - h_f\right) - \ Auxiliaries}{m_F q_F}\ x\ 100\%$$

Where $Auxiliaries$ represents the total energy rate consumed by parasitic auxiliary equipment, expressed in equivalent units of thermal energy per hour (kJ/hr).

HYDRO ELECTRIC POWER PLANT

A **hydro-electric power plant** is a renewable energy generation facility that converts the gravitational potential energy of accumulated water into kinetic energy, and subsequently into usable mechanical and electrical power. The conversion process begins at an elevated reservoir, where water is held behind a dam to establish a hydrostatic head. Water is then released through a pressurized conduit known as a **penstock**. As water descends through the penstock, its potential energy is converted into kinetic energy and pressure head, which directly drives the rotating runners of a hydraulic turbine. The mechanical torque generated by the turbine shaft rotates an electromagnetic generator rotor, producing high-voltage electricity.



The total energy available for conversion depends heavily on the hydraulic head, which is the vertical distance through which the water falls. In professional licensing examinations, it is critical to distinguish how this head is defined, measured, and applied across different classes of hydraulic turbines, specifically impulse turbines (such as the Pelton wheel) and reaction turbines (such as Francis and Kaplan turbines).

Gross Head (H_g) Characteristics and Reference Datums

The Gross Head (**H_g**), also referred to as the available head, is the total theoretical vertical distance between the free water surface level at the reservoir (headwater) and a specific lower reference datum. Because different turbine classes operate on different physical principles, the lower reference datum used to measure H_g varies according to turbine design.

1. Impulse Turbines (Pelton Turbine Configuration)

A Pelton turbine is an impulse turbine suited for high-head, low-flow applications. In an impulse system, high-pressure water from the penstock is completely expanded to atmospheric pressure through one or more stationary control nozzles before hitting the turbine blades. The high-velocity water jet strikes bowl-shaped buckets on the runner, transferring momentum.

Because the runner operates entirely in an open, unpressurized casing at atmospheric pressure, any vertical distance between the nozzle discharge and the lower tailrace pool cannot be utilized to perform work. Therefore, for a Pelton turbine, the lower reference datum for measuring the Gross Head (H_g) is the **horizontal centerline of the inlet nozzle**:

$$H_{g,\,Pelton} = Z_{headwater} - Z_{nozzle}$$

2. Reaction Turbines (Francis and Kaplan Turbine Configurations)

Francis and Kaplan turbines are reaction turbines used in medium-to-low head and high-flow applications. Unlike impulse turbines, reaction turbines run completely submerged in water within a closed, pressurized system. Water enters the runner with both pressure and kinetic energy, and its pressure drops continuously as it flows through the runner blades.

To prevent the loss of potential energy after the water exits the runner, reaction turbines use a diverging conduit called a **draft tube**. The draft tube connects the runner exit to the tailrace, keeping the entire water column sealed and continuous. This column of water creates a suction head at the runner outlet, allowing the turbine to utilize the entire vertical drop down to the tailrace level.

Consequently, for Francis and Kaplan turbines, the lower reference datum for measuring the Gross Head (H_g) is the **free surface level of the tailrace**:

$$H_{g,\,Reaction} = Z_{headwater} - Z_{tailrace}$$

Within this system, the elevation difference between the turbine runner centerline and the tailrace surface level is designated as the static draft head or suction height (Z_B). Maintaining the correct Z_B value is critical; if the runner is placed too high above the tailrace, the suction pressure at the runner exit can fall below the vapor pressure of water, causing vapor bubbles to form and collapse—a destructive phenomenon known as cavitation.

Thermodynamic and Hydraulic Energy Distribution

In a hydro-electric power plant, power generation is modeled as a continuous, step-down energy conversion process. The potential energy of water held at the elevated headwater reservoir is converted into kinetic energy and fluid pressure as it flows through the penstock, before finally being converted into mechanical shaft work and electrical energy. Evaluating this process for board examinations requires tracking the fluid's energy state at key physical boundaries within the system.

To determine the energy available at different stages of the system, we distinguish between two primary reference heads: the static gross head at the reservoir and the dynamic net head available at the turbine inlet nozzle.

Hydraulic Power at the Headwater (WP_{HW})

The maximum potential energy rate available at the source is defined by the Gross Head (H_g). This value represents the water power at the headwater (WP_{HW}), which is the theoretical power available before accounting for hydraulic friction and fitting losses:

$$WP_{HW} = Q\gamma_W H_g$$

Where:

- WP_{HW} = Hydraulic power at the headwater (W or kW)
- Q = Total volumetric flow rate entering the system (m^3/s)
- γ_W = Specific weight of water (N/m^3 or kN/m^3), where $\gamma_W = \rho_W\left(\frac{g_o}{g_c}\right)$
- H_g = Gross hydraulic head (m)

Net or Effective Head (H_e)

As water flows from the reservoir to the turbine, it experiences friction and turbulence losses. The net or effective head (H_e) is the actual physical head remaining at the turbine inlet nozzle after subtracting these cumulative energy losses (H_L):

$$H_e = H_g - H_L$$

$$H_e = H_g - (h_e + h_p + h_n)$$

Where the total head loss (H_L) is the sum of:

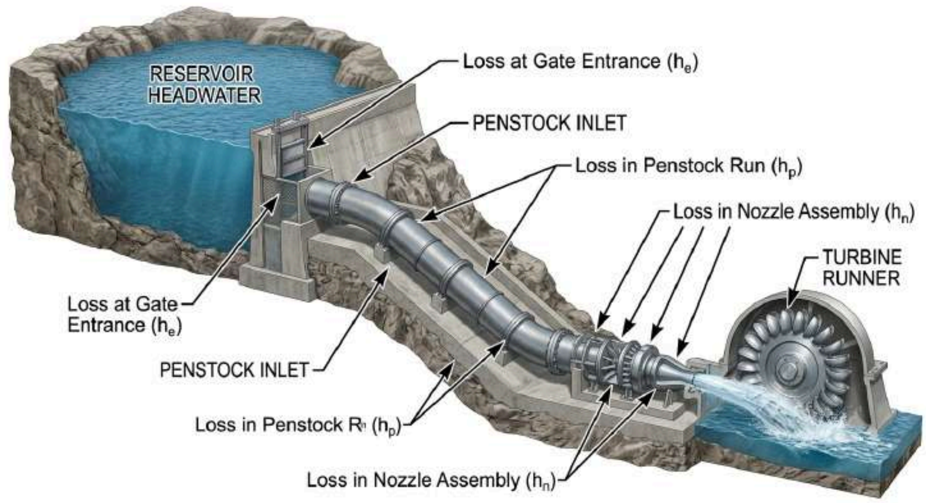
- h_e = Minor head loss at the intake gate entrance
- h_p = Major friction and minor losses along the penstock pipeline
- h_n = Hydraulic head loss within the nozzle assembly itself

Hydraulic Power at the Nozzle (WP)

The net, usable water power delivered directly to the turbine inlet nozzle (WP) is calculated using the remaining effective head:

$$WP = Q\gamma_W H_e$$

A series of thermodynamic, volumetric, and mechanical losses occur as water transfers its energy to the rotating turbine runner. These losses are evaluated using a continuous chain of efficiency metrics.



Penstock Efficiency (η_P)

Penstock efficiency measures the effectiveness of the pipeline in transporting fluid energy. It quantifies the ratio of water power delivered at the nozzle to the maximum potential power at the reservoir headwater, which simplifies to the ratio of their respective heads:

$$\eta_P = \frac{WP}{WP_{HW}} x\, 100\% = \frac{H_e}{H_g} x\, 100\%$$

Volumetric Efficiency (η_V)

In physical turbine systems, a small portion of the water bypasses the runner through clearance spaces, seals, and wear rings. This bypass flow represents a volumetric leakage loss (Q_L) that does not perform work on the runner blades. Volumetric efficiency (η_V) is the ratio of the power in the water that actually strikes the runner to the total water power delivered to the nozzle:

$$\eta_V = \frac{(Q - Q_L)\gamma_W H_e}{Q\gamma_W H_e} x\, 100\% = \frac{Q - Q_L}{Q} x\, 100\%$$

Where:

- Q_L = Volumetric leakage loss flow rate (m^3/s)
- $Q - Q_L$ = Net volumetric flow rate acting directly on the turbine blades

Hydraulic Efficiency (η_h)

Hydraulic efficiency measures the effectiveness of the runner blades in converting the fluid's kinetic and pressure energy into mechanical work. It is defined as the ratio of the mechanical power developed by the runner blades (Blade Power) to the net hydraulic power of the water acting on them:

$$\eta_h = \frac{Blade\ Power}{(Q - Q_L)\gamma_W H_e}\ x\ 100\%$$

Because the mechanical power transferred to the blades can be expressed using the utilized head (H_u), this equation simplifies to:

$$\eta_h = \frac{(Q - Q_L)\gamma_W H_u}{(Q - Q_L)\gamma_W H_e}\ x\ 100\% = \frac{H_u}{H_g}\ x\ 100\%$$

Where H_u represents the utilized head, which is the equivalent height of fluid energy successfully captured by the turbine buckets or runner blades.

Mechanical Efficiency (η_M)

Not all mechanical power developed by the runner blades is transferred to the generator. A portion is lost to friction in the bearings, packing glands, and windage. Mechanical efficiency $\left(\eta_M\right)$ is the ratio of the net power delivered at the turbine output shaft (Brake Power, BP) to the total power developed at the blades (Blade Power):

$$\eta_M = \frac{BP}{Blade\ Power}\ x\ 100\%$$

Where the mechanical energy balance is defined as:

Blade Power = BP + FP

Here, FP represents Friction Power, which is the rate of energy loss due to mechanical friction.

Overall Turbine Efficiency (η_T)

The overall turbine efficiency (η_T) evaluates the total performance of the turbine unit. It represents the ratio of the net mechanical shaft output (Brake Power, BP) to the total hydraulic power delivered to the nozzle (WP):

$$\eta_T = \frac{BP}{WP}\ x\ 100\%$$

This overall efficiency can also be calculated as the product of the individual volumetric, hydraulic, and mechanical efficiencies:

$$\eta_T = \eta_v \cdot \eta_h \cdot\ \eta_M$$

Generator Efficiency (η_g)

The driven generator converts mechanical shaft power (BP) into electrical output (EP). Generator efficiency (η_v or η_e) measures the electrical and magnetic losses during this conversion:

$$\eta_e = \frac{EP}{BP}\, x\, 100\%$$

Where EP represents the net electrical power output generated by the facility.

NET OR EFFECTIVE HEAD

To derive the exact expression for the effective head (H_e) across different turbine configurations, we apply the steady-flow energy equation (First Law of Thermodynamics) between the reservoir surface (Point A) and the turbine inlet/outlet boundary (Point B).

Assuming a steady, incompressible, and adiabatic flow, the energy balance per unit weight of fluid is written as:

$$z_A + \frac{v_A^{\,2}}{2g_C} + \frac{P_A}{\gamma_W} = z_B + \frac{v_B^{\,2}}{2g_C} + \frac{P_B}{\gamma_W} + H_L$$

Where:

- z = Elevation head relative to a reference datum
- $\frac{v^2}{2g_C}$ = Velocity (kinetic) head (g_C is the dimensional proportionality constant)
- $\frac{P}{\gamma_W}$ = Pressure head
- H_L = Cumulative head loss between boundaries

Net or Effective Head for Pelton Turbines

For an impulse Pelton turbine, the nozzle centerline serves as the reference datum ($z_B = 0$). The reservoir surface is open to the atmosphere ($P_A = 0$), and its large surface area means the surface velocity is negligible ($v_A = 0$). Applying these boundary conditions simplifies the energy equation to:

$$z_A - H_L = \frac{v_B^{\,2}}{2g_C} + \frac{P_B}{\gamma_W}$$

Because the net head H_e represents the total energy remaining in the fluid at the turbine inlet nozzle, the effective head for a Pelton turbine is:

$$H_e = \frac{v_B^{\,2}}{2g_C} + \frac{P_B}{\gamma_W}$$

Where v_B and P_B are the velocity and static pressure of the fluid measured immediately upstream of the nozzle inlet.

Net or Effective Head for Kaplan or Francis Turbines

In a reaction turbine system (Kaplan or Francis), the turbine runner is positioned at an elevation z_B above the tailrace reference datum ($Z_{tailrace}$ = 0). Applying the same reservoir boundary conditions ($P_A = 0$, $v_A = 0$), the energy equation yields:

$$z_A - H_L = z_B + \frac{v_B^{\,2}}{2g_C} + \frac{P_B}{\gamma_W}$$

Consequently, the net head available to a reaction turbine must account for both the pressure/kinetic energy at the inlet and the elevation head (z_B) relative to the tailrace:

$$H_e = z_B + \frac{v_B^{\,2}}{2g_C} + \frac{P_B}{\gamma_W}$$

Where z_B represents the physical height of the turbine runner inlet above the tailrace discharge surface.

Volumetric Flow Rate (Q) Through Control Nozzles

To calculate the real-time volumetric discharge through an impulse turbine control nozzle, we use the nozzle area and fluid velocity, adjusted for frictional losses using the nozzle discharge coefficient (C_d):

The theoretical velocity of the water jet exiting the nozzle (v_j) is calculated from the available head (h_j) acting on it:

$$v_j = \sqrt{2g_o h_j}$$

Where h_j is the net head at the nozzle inlet. The actual volumetric flow rate (Q) passing through a nozzle with exit area A_j is then defined as:

$$Q = C_d A_j v_j = C_d A_j \sqrt{2g_o h_j}$$

Where C_d is the dimensionless coefficient of discharge, which accounts for friction and flow contraction at the nozzle orifice.

Specific Speed of Hydraulic Turbines (N_s)

Specific speed (**N_s**) is a dimensionless parameter used to classify turbine runners based on their geometric proportions rather than their physical size. It represents the rotational speed of a geometrically similar turbine that would develop unit power under a unit head. This value is critical for selecting the optimal turbine type (Pelton, Francis, or Kaplan) for a given site's head and flow conditions.

Depending on the system of units used, specific speed is calculated using one of two standard formulations:

1. Specific Speed in Imperial Units (U.S. Customary System)

When head is measured in feet and power is measured in horsepower, specific speed is defined as:

$N_s = \frac{N\sqrt{HP}}{h^{5/4}}$	Where: • N_s = Specific speed (rpm) • N = Turbine rotational speed (rpm) • HP = Shaft power output (horsepower) • h = Net head (feet)

Specific Speed in Metric Units (SI System)

When head is measured in meters and power is measured in kilowatts, the specific speed calculation is adjusted to:

$N_s = 0.2623\frac{N\sqrt{kW}}{h^{5/4}}$	Where: • kW = Shaft power output (kilowatts) • h = Net head (meters)

Generator Synchronous Speed (N)

To produce electricity at a stable utility grid frequency, the turbine shaft must rotate at a constant synchronous speed. This speed (N) is determined by the grid frequency (f) and the physical number of magnetic poles (P) in the generator rotor:

$$N = \frac{120f}{P}$$

Where:

- N = Rotational speed of the generator rotor and turbine shaft (rpm)
- f = Grid frequency in cycles per second (Hz)
- P = Number of magnetic poles in the generator. To maintain magnetic symmetry, P must be an **even integer** (2, 4, 6, 8,...).