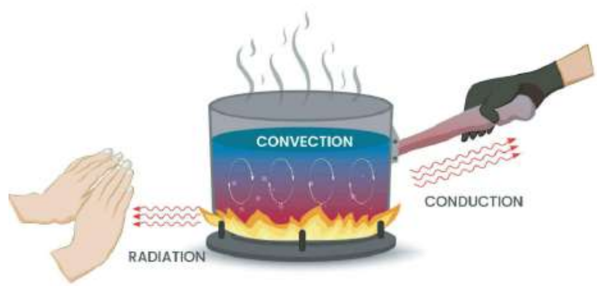
Heat Transfer

In the study of engineering thermodynamics and thermal sciences, heat transfer is defined as a specialized branch of science that deals with the rate of transfer of thermal energy between or within bodies due to the presence of a temperature difference. While classical thermodynamics focuses on systems at equilibrium states and the total quantity of energy required to transition from one equilibrium state to another, heat transfer addresses the specific mechanisms, governing laws, and kinetic rates at which this energy is exchanged over time. The fundamental driving force or potential for any heat transfer process is a spatial temperature gradient. In accordance with the Second Law of Thermodynamics, thermal energy spontaneously flows from a region of higher temperature to a region of lower temperature.

The Primary Modes of Heat Transfer

Thermal energy is transmitted across physical boundaries through three distinct, fundamental modes: conduction, convection, and radiation.



1. Conduction Heat Transfer

Conduction is the transport of thermal energy within a stationary medium or between two or more distinct bodies that are in direct physical contact with one another.

At the microscopic scale, conduction occurs due to the exchange of kinetic energy between atoms, molecules, or free electrons. In a high-temperature region, molecules possess higher kinetic energy and exhibit intense vibrational and translational motion. As these energetic particles collide with adjacent, less energetic particles in lower-temperature regions, a direct transfer of thermal energy takes place. In solids, this process is further enhanced by the lattice vibrational waves (phonons) and the drift of free electrons, which explains why materials with high electrical conductivity, such as metals, also exhibit high thermal conduction capabilities. The fundamental law governing this mode dictates that the heat flow is directly proportional to the temperature difference and the contact surface area.

2. Convection Heat Transfer

Convection heat transfer is not a purely independent mode, but rather a combination of microscopic conduction and the macroscopic transport of heat energy due to fluid circulating within the medium.

Convection occurs when a fluid (either a liquid or a gas) comes into motion relative to a solid surface or another fluid layer at a different temperature. The process begins with conduction at the immediate solid-fluid interface, where a stationary thin film of fluid layer directly touches the solid surface. Thermal energy is conducted into this initial layer, which alters its local density. As the bulk fluid circulates—either due to naturally occurring buoyancy forces arising from density differences (natural or free convection) or due to external mechanical forces such as pumps, fans, or wind (forced convection)—it sweeps the heated fluid molecules away and replaces them with cooler fluid, drastically accelerating the overall energy transfer rate.

3. Radiation Heat Transfer

Radiation is the transfer of thermal energy through the emission and propagation of electromagnetic radiation.

Unlike conduction and convection, which strictly demand the physical presence of a material medium to facilitate molecular interaction or fluid movement, radiation heat transfer requires no intervening material medium and can propagate with maximum efficiency through a perfect vacuum. All matter that possesses a finite temperature above absolute zero (0 K) continuously emits thermal radiation. This energy is transmitted in the form of electromagnetic waves (or photons) traveling at the speed of light. When these waves strike another surface, a portion of the incident energy is absorbed and converted back into internal thermal energy, establishing a radiative heat exchange between emitting and absorbing bodies.

Conduction through Plane and Composite Walls

Fundamental Principles of One-Dimensional Heat Conduction

Conduction heat transfer through solid mediums is mathematically governed by **Fourier's Law of Heat Conduction**. This law states that the rate of heat transfer through a homogeneous material is directly proportional to the perpendicular area through which the heat flows, the intrinsic thermal conductivity of the material, and the temperature gradient across the physical boundaries. When analyzing a single flat layer (a plane wall) under steady-state conditions, the temperature profile changes linearly with position, allowing the differential expression to be simplified to a macro-level algebraic formula:

$$Q = -kA\frac{\Delta T}{\Delta x}$$

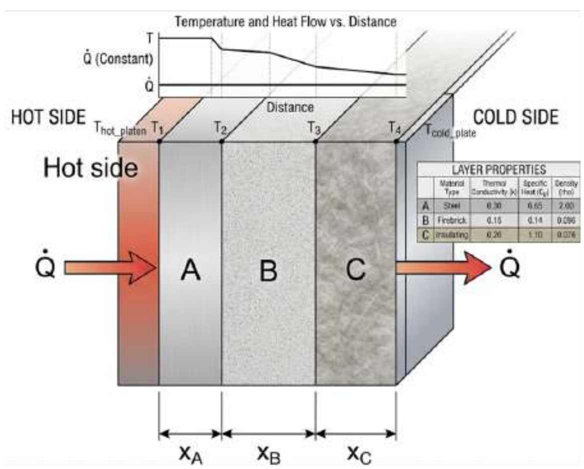
Where:

- Q = Rate of heat flow (W or Btu/h)
- k = Thermal conductivity of the material (W/m-ºC or Btu/h-ft-ºF)
- A = Perpendicular cross-sectional area through which heat passes (m^2 or ft^2)
- ΔT = Temperature difference driving the heat flow (T_{cold} - T_{hot})
- Δx = Physical thickness of the solid wall (m or ft)

The negative sign in the fundamental equation is a structural thermodynamic necessity. It ensures that heat flow remains mathematically positive in the direction of a negative temperature gradient (meaning heat naturally moves down the thermal gradient from a region of higher temperature to a region of lower temperature, matching the Second Law of Thermodynamics).

Steady-State Multi-Layer Composite Walls

In real-world industrial installations—such as furnace structural linings, insulated boiler casings, or building envelopes—multiple different material layers are joined tightly together to form a **Composite Wall**. When a system composed of three distinct materials (A, B and C) arranged in series reaches a steady-state condition, an essential principle applies: **the rate of heat flow (Q) must remain absolutely constant and uniform through every single section.**



Because energy cannot accumulate or be generated within any internal layer under steady conditions, the heat entering the first layer must exactly equal the heat passing through the intermediate boundaries and exiting the final cold face:

$$Q = -k_A A\frac{T_2 - T_1}{\Delta x_A} = -k_B A\frac{T_3 - T_2}{\Delta x_B} = -k_C A\frac{T_4 - T_3}{\Delta x_C}$$

By removing the negative signs and rearranging each individual expression to isolate the temperature drops across each specific material layer, we get:

$$T_1 - T_2 = Q\left(\frac{\Delta x_A}{k_A A}\right)$$

$$T_2 - T_3 = Q\left(\frac{\Delta x_B}{k_B A}\right)$$

$$T_3 - T_4 = Q\left(\frac{\Delta x_C}{k_C A}\right)$$

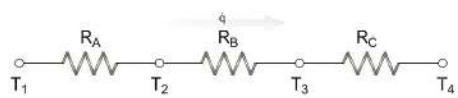
Solving these three structural equations simultaneously by adding them together cancels out the intermediate interface temperatures (T_2 and T_3), leaving a single, consolidated expression based entirely on the total overall temperature difference across the entire assembly:

$$T_1 - T_4 = Q\left(\frac{\Delta x_A}{k_A A} + \frac{\Delta x_B}{k_B A} + \frac{\Delta x_C}{k_C A}\right)$$

$$Q = \frac{T_1 - T_4}{\left(\frac{\Delta x_A}{k_A A} + \frac{\Delta x_B}{k_B A} + \frac{\Delta x_C}{k_C A}\right)}$$

3. The Electrical Analogy for Heat Conduction

To simplify the analysis of complex, multi-layered thermal systems, engineers rely on the **Electrical Analogy**. This framework maps thermal parameters directly to the classical equations of electrical direct-current (DC) circuits governed by Ohm's Law (I=V/R).



By matching variables, the fundamental master equation can be structurally redefined as:

$$Q = \frac{Thermal\ Potential\ Difference}{Total\ Thermal\ Resistance} = \frac{\Delta T_{overall}}{R_{total}}$$

- **Thermal Potential Difference (ΔT):** Corresponds directly to an electrical voltage potential (V). It represents the thermal driving force created by the temperature disparity across the boundaries.
- **Heat Flow Rate (Q):** Corresponds to electrical current (I). It tracks the rate of kinetic energy transmission through the system.
- **Convective/Conductive Thermal Resistance (R):** Corresponds to an electrical resistor (R). For conduction through a flat plane wall, the individual internal resistance is explicitly defined as:

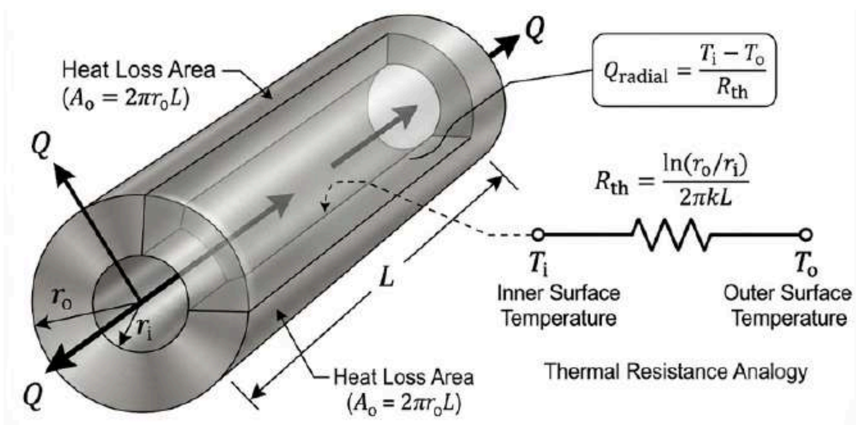
$$R_{cond} = \frac{\Delta x}{kA} \quad ; \quad \text{units: } \frac{^oC}{W} \text{ or } \frac{^oF-hr}{Btu}$$

Because the material slabs A, B and C are arranged sequentially back-to-back, their individual thermal resistances act in a strict **series network layout**. Consequently, the total composite thermal resistance is found by calculating the direct sum of all individual layer resistances:

$$R_{total} = R_A + R_B + R_C = \frac{\Delta x_A}{k_A A} + \frac{\Delta x_B}{k_B A} + \frac{\Delta x_C}{k_C A}$$

One-Dimensional Radial Heat Conduction

When analyzing heat transfer in industrial piping, distribution tubes, or lagged electrical cables, the plane wall assumption is no longer dimensionally accurate. These geometries fall under the category of **Radial Systems**, where thermal energy flows in a direction perpendicular to the system's central longitudinal axis. For a steady-state hollow cylinder of length L, an inner radius r_i, and an outer radius r_o, heat transfer is treated as one-dimensional if temperature varies only in the radial direction (r).



Unlike plane walls where the cross-sectional area remains constant along the path of conduction, the surface area through which heat passes in a radial system increases continuously from the inner boundary to the outer surface. This area at any radius r within the cylinder wall is defined by the geometric formula:

$$A = 2\pi rL$$

Because this cross-sectional area changes dynamically as a function of the radius, the rate of temperature decrease is non-linear. Incorporating this expanding area into Fourier's Law of Heat Conduction creates a differential equation that, upon integration across the radial boundaries, yields the mathematical model for heat transfer through cylindrical geometries.

Steady-State Conduction through a Hollow Cylinder

Under steady-state conditions with no internal heat generation, the total rate of radial heat flow (Q) passing through any concentric layer of the cylinder must remain constant. Applying the electrical network analogy introduces a streamlined path to solve these systems. The master heat equation states that the heat flow rate is equal to the overall thermal potential difference divided by the summation of the network's thermal resistance:

$$Q = \frac{\Delta T_{overall}}{\Sigma R_{th}}$$

For conduction through a single hollow cylinder layer, the isolated thermal circuit consists of an inner surface temperature (T_i) at the inner radius (r_i) and an outer surface temperature (T_o) at the outer radius (r_o).

Integrating the expanding surface area profile yields the specific expression for the **Cylindrical Thermal Resistance (R_{th})**:

$$R_{th} = \frac{ln\left(\frac{r_o}{r_i}\right)}{2\pi rkL}$$

Where:

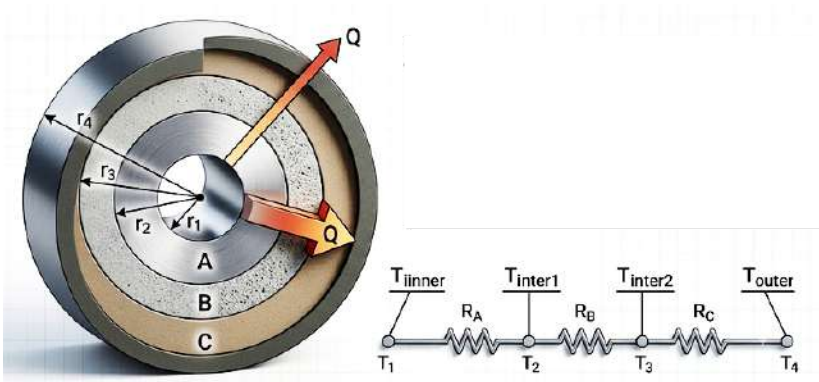
- ln = The natural logarithm function
- r_o = Outer radius of the cylinder
- r_i = Inner radius of the cylinder
- k = Thermal conductivity of the cylinder material
- L = Axial length of the hollow cylinder

Substituting this precise geometric resistance value back into the electrical analogy formula yields the final integrated equation for steady-state radial heat flow:

$$Q = \frac{2\pi rkL\left(T_i - T_o\right)}{ln\left(\frac{r_o}{r_i}\right)}$$

Conduction through Multiple Cylindrical Sections (Composite Pipes)

In complex industrial applications, a single cylindrical layer is rarely sufficient to meet thermal management demands. For instance, a high-temperature steam pipe typically features a metallic wall surrounded sequentially by layers of refractory material, insulation wool, and a protective outer weather jacket. This arrangement is structurally classified as a **multiple cylindrical section** or a multi-layer composite radial system.



When analyzing a three-layered cylindrical system consisting of materials A, B, and C, heat transfer operates under steady-state conditions with no internal heat generation. Under these parameters, the total rate of radial heat flow (Q) passing through each individual layer is identical and constant:

$$Q = \frac{\Delta T_{overall}}{\Sigma R_{th}}$$

Using the electrical analogy for series networks, the intermediate temperatures (T_2 and T_3) at the material interfaces adjust dynamically until a completely uniform energy flow rate is achieved across the entire network. The individual conductive thermal resistances for each consecutive layer are calculated based on their unique material thermal conductivities (k_A, k_B, k_C) and their bounding geometric radii (r_1 to r_4):

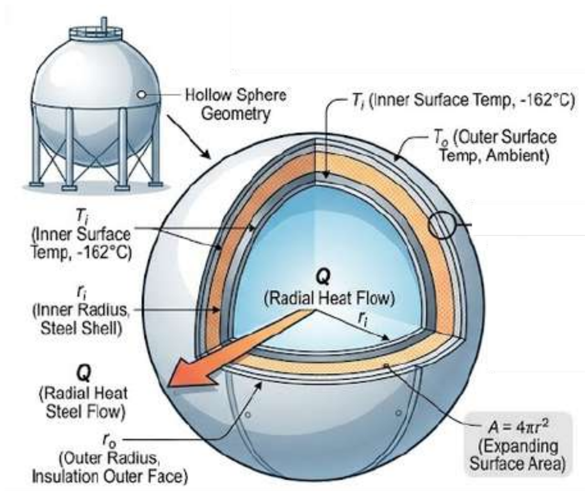
$$R_{th} = \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi r k_A L} + \frac{ln\left(\frac{r_3}{r_2}\right)}{2\pi r k_B L} + \frac{ln\left(\frac{r_4}{r_3}\right)}{2\pi k_C k L}$$

Factoring out the shared geometric parameter $2\pi L$ from the denominator of each resistance term allows the steady-state heat transfer equation for a composite cylinder to be consolidated into a single calculation:

$$Q = \frac{2\pi r L\left(T_1 - T_4\right)}{\frac{ln\left(\frac{r_2}{r_1}\right)}{k_A} + \frac{ln\left(\frac{r_3}{r_2}\right)}{k_B} + \frac{ln\left(\frac{r_4}{r_3}\right)}{k_C}}$$

Conduction through Spherical Systems

While cylindrical models describe long pipelines, certain engineering systems—such as cryogenic liquefied natural gas (LNG) storage tanks, spherical pressure vessels, and chemical reactors—require a spherical framework.



A **Hollow Sphere** with an inner radius r_i and an outer radius r_o presents a unique geometric condition. As heat moves radially outward from the center, the perpendicular surface area increases continuously. This expanding area varies as a function of the square of the radius ($A = 4\pi r^2$). Integrating Fourier's Law across these boundaries yields the structural **Spherical Thermal Resistance (R_{th})** expression:

$$R_{th} = \frac{1}{4\pi r k r_i} + \frac{1}{4\pi r k r_o}$$

By finding a common denominator for the radius terms, this resistance formula can be algebraically rearranged into a single fraction:

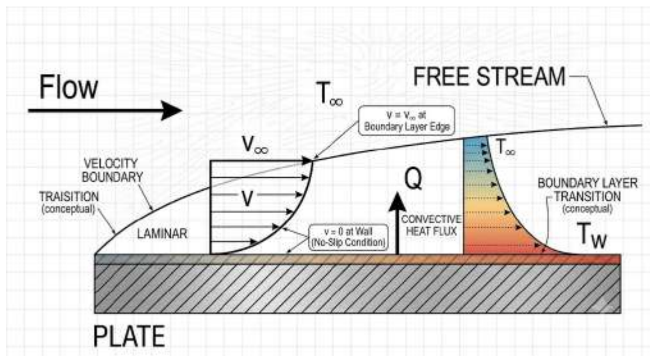
$$R_{th} = \frac{r_o - r_i}{4\pi r k r_i r_o}$$

Substituting this spherical resistance back into the master electrical analogy formula $\left(Q = \frac{\Delta T}{R_{th}}\right)$ provides the steady-state radial heat transfer equation for a hollow sphere:

$$Q = \frac{4\pi k\left(T_i - T_o\right)}{\frac{1}{r_i} + \frac{1}{r_o}}$$

Convection Heat Transfer

In real-world thermal applications, heat transfer rarely occurs via a single isolated mode. When analyzing systems where a solid boundary interacts with a moving fluid, the process is fundamentally categorized as **Convection Heat Transfer**. Consider a fluid passing over a heated horizontal plate with a surface temperature designated as T_w (wall temperature) while the bulk temperature of the circulating free-stream fluid is maintained at T_{∞}. As the fluid sweeps across the surface, a velocity boundary layer forms where fluid velocity variations (v) scale up from zero at the wall up to the free-stream velocity (V^{∞}).



The kinetics governing this boundary layer are described by **Newton's Law of Cooling**. This physical principle states that the rate of heat flow from an object is directly proportional to the difference between its own surface temperature and the temperature of the surrounding circulating fluid. Mathematically, this relation is modeled as:

$$Q_{conv} = hA\left(T_w - T_{\infty}\right)$$

Where:

- Q_{conv} = Rate of convective heat transfer (W or Btu/h).
- A = Heat flow surface area (m^2 or ft^2).
- H = Convection heat transfer coefficient, commonly referred to as **film conductance**. Its standard units are expressed as $\frac{W}{m^2 - \circ C}$ in SI, or $\frac{Btu}{h - ft^2 - \circ F}$ in USCS.

To maintain analytical consistency with standard circuit analysis, an **electrical-resistance analogy** can be drawn for the convection process. By isolating the rate parameters, the convective heat transfer expression can be structurally reconfigured as a potential-to-resistance ratio:

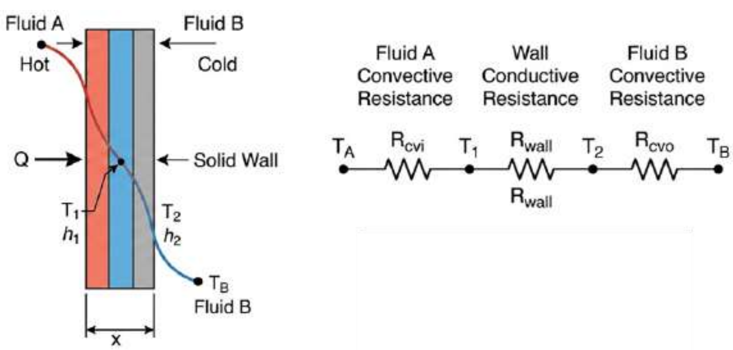
$$Q = \frac{Thermal\ Potential\ Difference}{Convection\ Thermal\ Resistance} = \frac{T_w - T_{\infty}}{\frac{1}{hA}}$$

From this structural derivation, the standalone value for **convective thermal resistance (R_{conv})** is formally defined as:

$$R_{conv} = \frac{1}{hA}$$

The Overall Heat Transfer Coefficient (U) for Plane Walls

When evaluating a complete industrial system—such as an insulated building wall or a heat exchanger shell—thermal energy must first pass from a hot fluid to a solid wall via convection, pass through the wall thickness via conduction, and subsequently exit into a cold fluid via a final convective boundary layer. Consider a solid plane wall of thickness Δx and thermal conductivity k, where one side is exposed to a hot fluid (Fluid A at bulk temperature T_A with film coefficient h_1) and the opposite face is in contact with a cooler fluid (Fluid B at bulk temperature T_B with film coefficient h_2).



Because these three thermal barriers are encountered sequentially in a steady-state line, they form a strict series resistance network. The overall rate of heat transfer through this combined path is defined by dividing the overall temperature drop by the sum of the convective and conductive resistances:

$$Q_{conv} = \frac{T_A - T_B}{\frac{1}{h_1 A} + \frac{\Delta x}{kA} + \frac{1}{h_3 A}}$$

To simplify extensive design layouts, engineers combine these distinct resistance parameters into a single lumped variable called the **Overall Heat Transfer Coefficient (U)**. This factor represents the total reciprocal thermal resistance per unit area:

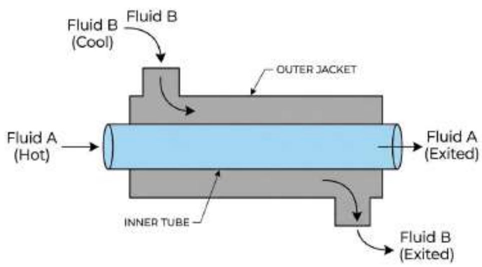
$$U = \frac{1}{\frac{1}{h_1 A} + \frac{\Delta x}{kA} + \frac{1}{h_3 A}}$$

Once Y is established, the general master expression for multi-mode steady heat flow through a flat profile simplifies to:

$$Q = UA\Delta T_{overall}$$

Combined Radial Systems: Cylindrical Heat Exchangers

The structural calculation changes significantly when moving from flat surfaces to radial geometries, such as a **hollow cylinder** exposed simultaneously to internal and external convective fluid streams. In equipment like double-pipe heat exchangers, a hot inner fluid (Fluid A) moves axially down the tube core, while a cooler external fluid (Fluid B) circulates over the outer jacket.



A critical distinction in radial systems is that **the convective surface area is not the same for both fluids**. Because the surface areas depend directly on the specific boundary radius, the inner convective layer relies on the internal diameter while the outer convective layer tracks the expanded external profile:

- Inner Convective Surface Area (A_i) = $2\pi\, r_i L$
- Outer Convective Surface Area (A_o) = $2\pi\, r_o L$

Compiling the inner convection, cylindrical wall conduction, and outer convection steps yields a three-part series resistance model. The steady-state heat loss through a hollow cylinder is given by:

$$Q = \frac{T_A - T_B}{\frac{1}{h_i A_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{kA} + \frac{1}{h_o A_o}}$$

Where:

- T_A and T_B = Bulk temperatures of Fluid A and Fluid B, respectively.
- h_i and h_o = Inside and outside convection film coefficients.
- r_i and r_o = Inner and outer radii of the containment tube.
- L = Total axial length of the cylindrical pipe segment.

Thermal Radiation

Unlike conduction and convection, which require a physical solid, liquid, or gaseous medium to facilitate energy transmission, **Radiation Heat Transfer** refers to the emission of energy via electromagnetic waves. This allows thermal radiation to propagate through a perfect vacuum. Every object with a temperature above absolute zero continuously emits thermal radiation due to the rapid microscopic movements and electromagnetic changes within its molecules.

Radiation Emitted by an Ideal Black Body

The foundation of modern radiation analysis is the **Stefan-Boltzmann Law of thermal radiation**, which sets a theoretical upper boundary for energy emission from an object. An idealized surface that absorbs all incoming radiation and emits the absolute maximum possible thermal energy at any given wavelength is called a **black body**. The total rate of heat energy emitted from a perfect black body is directly proportional to its surface area and the fourth power of its absolute thermodynamic temperature. The mathematical model is expressed as:

$$Q = \sigma A T^4$$

Where:

- Q = Rate of thermal radiation emission (W or Btu/h).
- A = Emitters total surface area (m^2 or ft^2).
- T = Absolute thermodynamic temperature (K or °R).
- σ = The **Stefan-Boltzmann constant**, a universal physical constant with empirical values defined as:

$$\sigma = 5.669 \, x \, 10^{-8} \, \frac{W}{m^2 - K^4} = 0.1714 \, x \, 10^{-8} \, \frac{Btu}{h - ft^2 - {}^\circ R^4}$$

Radiation Interactions for Real Systems (Gray Bodies)

Real physical objects do not radiate energy as efficiently as a theoretical black body. To analyze real-world materials, engineers utilize a model known as a **gray body**. A gray body modifies ideal black body predictions by introducing a surface property factor known as **emissivity (ϵ)**. Emissivity measures an object's efficiency at emitting thermal energy compared to a perfect black body at identical temperatures:

$$\epsilon = \frac{Q_{real}}{Q_{black\ body}}$$

The numerical value of ϵ stays strictly between 0 and 1 for real surfaces, scaling exactly to 1 when evaluating an ideal black body. When analyzing the net steady-state heat exchange between a gray body at temperature T_1 and its surrounding gray enclosure at temperature T_2, the corrected mathematical relation is modeled as:

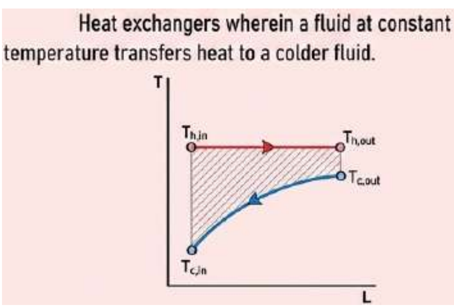
$$Q = \epsilon \sigma A \left(T_1^{\ 4} - T_2^{\ 4} \right)$$

Heat Exchangers

In industrial and power engineering systems, thermal management often requires the redistribution of internal energy from one system fluid to another. A **heat exchanger** is a specialized mechanical device designed to efficiently transfer thermal energy from a high-temperature fluid (hot stream) to a lower-temperature fluid (cold stream) across a separating physical barrier. The primary operating constraint of this device is that it must facilitate this rapid thermal energy migration without allowing the two working fluids to physically touch, mix, or contaminate each other. These systems are essential to operations in power generation plants, chemical refineries, HVAC systems, and automotive propulsion loops.

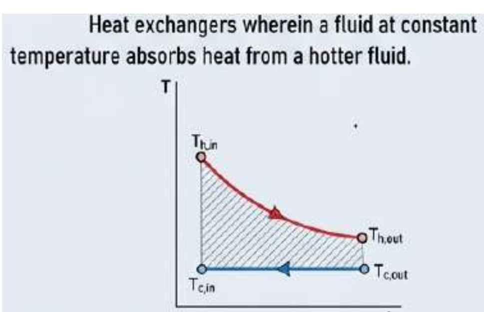
Heat exchangers are categorized into distinct classes based on the thermodynamic state behavior of the fluids or the geometric flow profiles governing the passage paths:

Class 1: Condensers (Constant Temperature Heat Source)



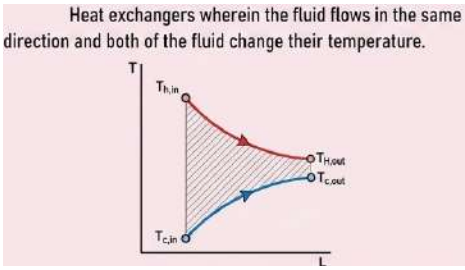
This class consists of heat exchangers where the hot fluid remains at a constant temperature throughout the entire length of the device while transferring heat to a colder fluid. Thermodynamically, this constant temperature line ($T_{h,in}=T_{h,out}$) indicates that the hot fluid is undergoing a phase change (condensation). As the vapor releases its latent heat of vaporization, the temperature of the colder fluid steadily rises exponentially from its inlet state ($T_{c,in}$) up to its outlet state ($T_{c,out}$) as it moves down the length (L) of the exchanger.

Class 2: Evaporators / Boilers (Constant Temperature Heat Sink)



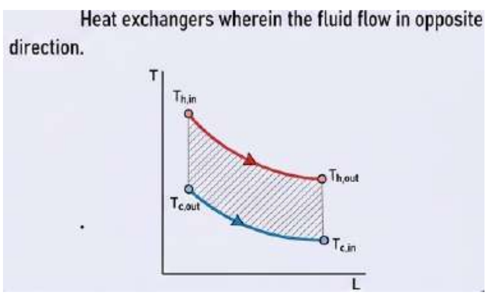
This class describes heat exchangers where a fluid remains at a constant temperature while absorbing heat from a hotter fluid. In this operational profile, the cold fluid is undergoing a phase change (boiling or evaporation), meaning its temperature line remains completely flat ($T_{c,in}=T_{c,out}$). Concurrently, the sensible heat given up by the non-phase-changing hot fluid causes its temperature to drop exponentially from its inlet value ($T_{h,in}$) down to its outlet value ($T_{h,out}$) over the device length (L).

Class 3: Parallel Flow Heat Exchangers



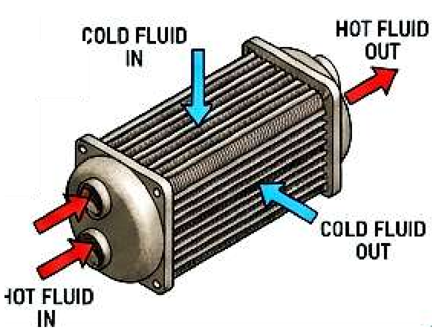
This configuration defines heat exchangers where both the hot and cold fluids enter the device at the same side, flow parallel to each other in the exact same physical direction, and exit on the same opposite side. As a result, both fluids change their temperatures simultaneously across the device length (L). The temperature profile reveals that the hot fluid drops rapidly from Th,in while the cold fluid rises rapidly from Tc,in. Because their temperature curves approach each other asymptotically, the exit temperature of the cold fluid ($T_{c,out}$) can never exceed the exit temperature of the hot fluid ($T_{h,out}$).

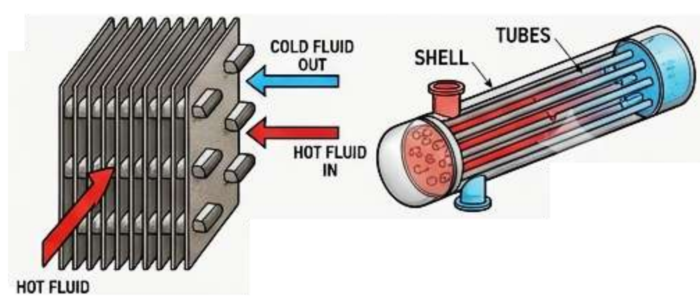
Class 4: Counter-Flow Heat Exchangers



This class describes heat exchangers where the two working fluids enter at opposite ends and travel in entirely opposite directions relative to one another. The hot fluid enters at one side ($T_{h,in}$) and cools as it flows toward its exit point ($T_{h,out}$). Meanwhile, the cold fluid enters from the opposite end ($T_{c,in}$) and warms up as it travels toward its respective exit ($T_{c,ou}$t). This counter-current positioning maintains a more uniform temperature difference along the entire length (L) of the exchanger, enabling the highest thermal effectiveness among all classes. It even allows the outlet temperature of the cold fluid to surpass the outlet temperature of the hot fluid.

Class 5: Cross-Flow Heat Exchangers



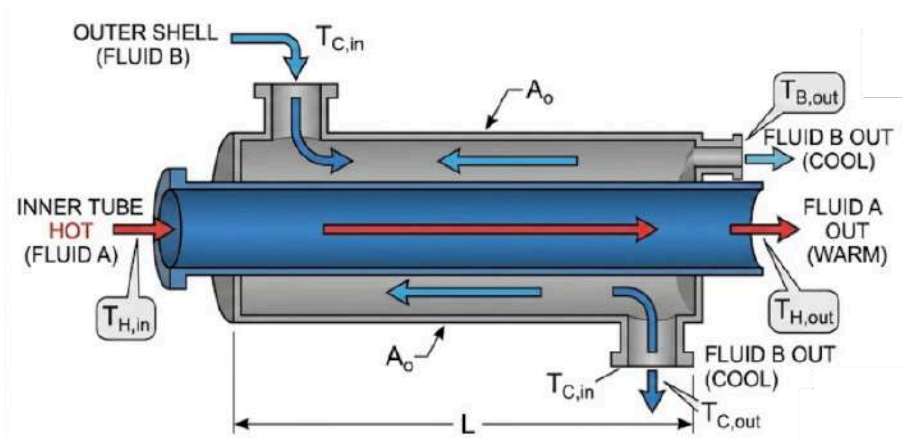


This class covers heat exchangers where the two fluids flow essentially perpendicular to each other. As shown in the visual schematic, one fluid (typically a liquid like a hot coolant) travels through the interior channels of a finned tube assembly, while the second fluid (typically a gas like ambient air) is forced crosswise through the exterior fins. This design is widely used in compact applications, such as automotive radiators and air conditioning coils, where a large surface area is required to maximize heat transfer to a gas stream.

- **$T_{h,in}$ / $T_{h,out}$:** The absolute temperatures of the hot fluid entering and exiting the heat exchanger bounds, respectively.
- **$T_{c,in}$ / $T_{c,out}$:** The absolute temperatures of the cold fluid entering and exiting the heat exchanger bounds, respectively.
- **L:** The linear coordinates measuring the cumulative length of the fluid interaction path inside the equipment shell.

Thermal Analysis of Double-Pipe Heat Exchangers

When evaluating thermal performance in a double-pipe heat exchanger, the temperatures of both fluids change continuously as they move along the length of the device.



Because the temperature driving force varies at every cross-section, calculating the total steady-state heat transfer rate (Q) requires using an integrated mean temperature difference across the entire heat exchange surface area (A). The general heat transfer rate equation is modeled as:

$$Q = UA\Delta T_m$$

Where:

- Q = Total rate of heat transfer.
- U = Overall heat-transfer coefficient.
- A = Surface area for heat transfer, which must be mathematically consistent with the geometric definition used to determine U.
- ΔT_m = The suitable mean temperature difference across the heat exchanger.

The Log Mean Temperature Difference (LMTD)

The standard mathematical approach for heat exchanger design and performance analysis is the **Log Mean Temperature Difference (ΔT_m or LMTD)**. Stated verbally, the LMTD is the temperature difference at one terminal side of the heat exchanger minus the temperature difference at the other terminal side, divided by the natural logarithm (ln) of the ratio of these two temperature differences.

> **Note:** The specific algebraic arrangement of the terminal temperature differences in the formula depends on whether the configuration is parallel flow or counter-flow. The equation provided below shows a generic layout corresponding to the standard mathematical structure:

$$\Delta T_m = \frac{\left(T_{H,\,out} - T_{C,\,out}\right) - \left(T_{H,\,in} - T_{C,\,in}\right)}{ln\left[\frac{T_{H,\,out} - T_{C,\,out}}{T_{H,\,in} - T_{C,\,in}}\right]} = LMTD$$

Where:

- $T_{H,\,in}$ / $T_{H,\,out}$ = Inlet and outlet temperatures of the hot fluid stream, respectively.
- $T_{C,\,in}$ / $T_{C,\,out}$ = Inlet and outlet temperatures of the cold fluid stream, respectively.

3. The Arithmetic Mean Temperature Difference (AMTD)

In certain preliminary design stages or when the temperature changes of both fluids are relatively small, engineers use a simplified linear approximation known as the **Arithmetic Mean Temperature Difference (ΔT_a or AMTD)**. This approximation treats the temperature profiles along the exchanger as linear paths rather than exponential curves. The arithmetic mean temperature difference is mathematically defined as the average of the terminal temperature differences at both ends of the exchanger:

$$\Delta T_a = \frac{\left(T_{H,\,out} - T_{C,\,out}\right) - \left(T_{H,\,in} - T_{C,\,in}\right)}{2} = AMTD$$

Heat Exchanger Efficiency / Effectiveness (ε)

When the fluid outlet temperatures are unknown, the log mean method requires an iterative, trial-and-error approach. To solve these problems directly, engineers use the **Effectiveness (ε)** method (frequently designated as heat exchanger efficiency). This dimensionless parameter measures the ratio of the actual temperature change of the fluid with the lower heat capacity rate to the maximum possible temperature difference across the entire heat exchanger:

$$\varepsilon = \frac{\Delta T\ (minimum\ fluid)}{Maximum\ temperature\ difference\ in\ heat\ exchanger}$$

The terms in this relationship are defined as follows:

- $\Delta T\ (minimum\ fluid)$**:** The actual temperature change ($\Delta T\ = T_{out} - T_{in}$) experienced by the fluid that possesses the smaller mass flow rate and specific heat product ($C_{min} = mC_p$). Because this fluid changes temperature more rapidly, it acts as the thermal constraint of the system.
- **Maximum Temperature Difference:** The absolute largest thermal potential driving force existing inside the equipment, which is always equal to the difference between the inlet temperature of the hot fluid and the inlet temperature of the cold fluid ($T_{H,\,in}\ -\ T_{C,\,in}$).

LMTD vs. AMTD Mathematical Reality: Mathematically, the AMTD is always greater than the LMTD (AMTD > LMTD). Consequently, using the AMTD to size a heat exchanger will overestimate the average driving temperature difference, leading to an under-designed, dangerously small surface area (A). Only rely on the AMTD approximation when the terminal temperature differences are very close to each other.

Always remember that the denominator for the heat exchanger effectiveness equation uses **only inlet temperatures** ($T_{H,\,in}\ -\ T_{C,\,in}$). This represents the maximum ideal thermal span that could be achieved if the exchanger had an infinite length in a counter-flow arrangement.