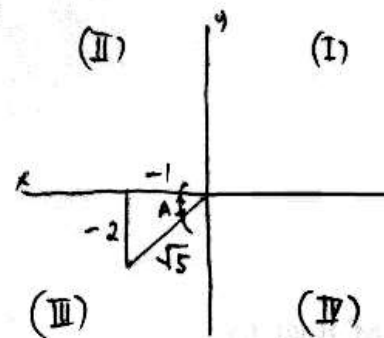


Answer Key

ENGINEERING MATHEMATICS

Refresher 03

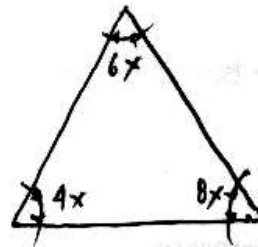




$$\cos A = \frac{1}{\sec A} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\sec A = \frac{\sqrt{5}}{-1}$$

$$\sec A = -\sqrt{5}$$



$$4x + 6x + 8x = 180^\circ$$

$$x = 10^\circ$$

Note: $1 \text{ rev} = 360^\circ = 6400 \text{ mills}$

Smallest angle:

$$4x = (40^\circ) \left(\frac{6400 \text{ mills}}{360^\circ} \right)$$

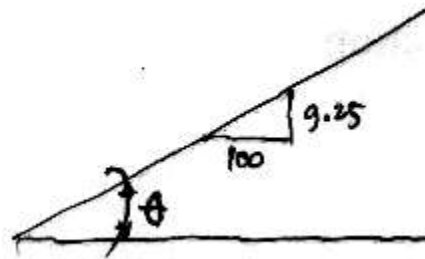
$$\text{Smallest Angle} = 711.111 \text{ mills}$$

3



Percentage

4



$$\tan \theta = \frac{9.25}{100}$$

$$\theta = 5.2848^\circ$$

$$A_1 - A_2 = 8 \text{ ft}^2$$

$$P_1 - P_2 = 5 \text{ ft}$$

for square: $b_1 = b_2$

$$b_1^2 - b_2^2 = 8 \text{ ft}^2 \longrightarrow (1) \quad 4b_1 - 4b_2 = 5 \text{ ft}$$

$$b_1 = \frac{5 + 4b_2}{4} \longrightarrow (2)$$

Subst. (2) to (1)

$$\left(\frac{5 + 4b_2}{4}\right)^2 - b_2^2 = 8 \text{ ft}^2$$

$$b_2 = 2.575 \text{ ft}$$

$$b_1 = 3.825 \text{ ft}$$

Hence; sum of areas:

$$A_1 + A_2 = (3.825 \text{ ft})^2 + (2.575 \text{ ft})^2$$

$$A_1 + A_2 = 21.26125 \text{ ft}^2$$

6

180 deg

7



Sector

8



Axis

M_L = male population literate

F_L = female population literate

$$F + M = 1500$$

$$F = 1500 - M \quad \leftarrow (1)$$

$$F_L + M_L = (0.55)(1500) \quad \text{"literate"}$$

$$F_L + M_L = 825 \quad \leftarrow (2)$$

$$F_L = 0.75F$$

$$M_L = 0.60M$$

Hence;

$$0.75F + 0.60M = 825$$

$$0.75(1500 - M) + 0.60M = 825$$

$$M = 2000$$

$$CC = C_0 + P_0/m + \frac{C_0 - C_L}{(1+i)^n - 1}$$

$$= \text{P } 300,000 + \frac{\text{P } 8,000}{0.08}$$

$$CC = \text{P } 400,000$$

||



Algebra

12



Helix

13



Corrosion

$$e_{i_{\text{Annualy}}} = e_{i_{\text{semi-annualy}}}$$

$$(1+i)^1 = \left(1 + \frac{0.06}{2}\right)^2$$

$$\bar{i} = 6.09\%$$

$$P = \frac{A}{i}$$

$$₱ 30,000 = \frac{A}{0.0609}$$

$$A = ₱ 1,827$$

$$e_{i \text{ monthly}} = e_{i \text{ annual}}$$

$$(1+i)^{12} = (1+0.05)^1$$

$$i = 4.0741 \times 10^{-3}$$

$$i = 0.40741\%$$

$$P = A \left[\frac{1 - (1+i)^{-N}}{i} \right]$$

$$₱120,000 = A \left[\frac{1 - (1.004074)^{-(12)(9)}}{4.0741 \times 10^{-3}} \right]$$

$$A = ₱1,375.6515$$

$$F = P(1+i)^N$$
$$= ₱9,000 \left(1 + \frac{0.17}{12}\right)^{(12)(7/12)}$$

$$F = ₱9,931.3396$$

17



Pitching

18



4 deg C

19



It needs 0.42 calorie to warm it by 1 deg

$$e_i = 27.54\% \text{ compounded monthly}$$

$$e_{\text{monthly}} = e_{\text{quarterly}}$$

$$0.2754 = \left(1 + \frac{i_n}{m}\right)^4 - 1$$

$$i = \frac{i_n}{m} = 0.06270 \text{ quarterly}$$

$$P = A \left[\frac{1 - (1+i)^{-N}}{i} \right]$$

$$P = \$26,000 \left[\frac{1 - (1.06270)^{-(12)(5)}}{0.06270} \right]$$

$$P = \$403,881.9648$$

$$I = F - P$$

$$F = P(1+i)^N = ₱1,200(1+0.09)^{17} = ₱5,193.1601$$

$$I = ₱5,193.1601 - ₱1,200$$

$$I = ₱3,993.1601$$

$$P = A \left[\frac{1 - (1+i)^{-N}}{i} \right]$$
$$= ₱530.22 \left[\frac{1 - (1.01142)^{-360}}{0.01142} \right]$$

$$P = ₱45,650.2527$$

$$P = A \left[\frac{1 - (1+i)^{-N}}{i} \right]$$

$$= ₱12,000 \left[\frac{1 - (1.083)^{-18}}{0.083} \right]$$

$$P = ₱110,159.8929$$

$$P = A \left[\frac{1 - (1+i)^{-N}}{i} \right]$$

$$₱632,000 = A \left[\frac{1 - (1.0095)^{-36}}{0.0095} \right]$$

$$A = ₱20,810.7814$$

25



Pascal's Principle

26



Ethical and legal are the same action

$$F = A \left[\frac{(1+i)^N - 1}{i} \right]$$

$$P20,000 = P1,300 \left[\frac{(1.11)^N - 1}{0.11} \right]$$

$$N = 9.4902 \text{ years}$$

$$N \approx 9.5 \text{ years}$$

$$I = 0.06P$$

$$F = P + I = P + 0.06P$$

$$F = P(1.06)$$

from:

$$F = P(1+i)^N$$

$$P(1.06) = P(1+i)^{(12)(\frac{1}{2})}$$

$$i = 0.9759\%$$

$$F = A \left[\frac{(1+i)^N - 1}{i} \right]$$

$$P700,000 = A \left[\frac{(1+0.0096)^{(6 \times 12)} - 1}{0.0096} \right]$$

$$A = \text{₱}6,297.7648$$

$$C_0 = ₱1,200,000$$

$$L = 7 \text{ years}$$

$$C_L = ₱110,000$$

Find; C_5

$$d = \frac{C_0 - C_L}{L} = \frac{₱1,200,000 - ₱110,000}{7}$$

$$d = ₱155,714.2857$$

then;

$$C_5 = C_0 - 5(d)$$

$$C_5 = ₱1,200,000 - 5(₱155,714.2857)$$

$$C_5 = ₱421,428.5714$$

$$1997 \quad C_0 = \text{P}900,000$$

$$L = 5 \text{ years}$$

$$C_L = \text{P}92,400$$

using: SYDM

$$d_n = \frac{\text{Reversed Digit}}{\text{Sum of Digits}} (C_0 - C_L)$$

$$1997 \quad 1 \quad 5$$

$$1998 \quad 2 \quad 4$$

$$1999 \quad 3 \quad 3$$

$$2000 \quad 4 \quad 2$$

$$2001 \quad 5 \quad 1$$

$$\text{Reversed Digit} = 3$$

$$\text{or using: Reversed Digit} = L - n + 1$$

$$\text{Sum of Digits} = \frac{L}{2} (L + 1) = \frac{5}{2} (5 + 1) = 15$$

$$d_3 = \frac{3}{15} (\text{P}900,000 - \text{P}92,400)$$

$$d_3 = \text{P}161,520$$

$$\begin{aligned}
 L &= 5 \text{ years} \\
 C_0 &= \text{P}550,000 \\
 C_L &= (0.35)(\text{P}550,000) = \text{P}192,500 \\
 \text{Inflation} &= i_f = 12\% \\
 \text{Appreciation} &= i_a = 12\%
 \end{aligned}$$

Due to inflation

$$\begin{aligned}
 C_0' &= C_0 (1 + i_f)^n \\
 &= \text{P}550,000 (1.12)^5 \\
 C_0' &= \text{P}969,287.9258
 \end{aligned}$$

Due to appreciation

$$\begin{aligned}
 C_L' &= C_L (1 + i_a)^n \\
 &= \text{P}192,500 (1.12)^5 \\
 C_L' &= \text{P}339,250.774
 \end{aligned}$$

$$\text{Replacement Cost} = C_0' - C_L'$$

$$= \text{P}969,287.9258 - \text{P}339,250.774$$

$$\text{Replacement Cost} = \text{P}630,037.1518$$

RATE OF return earned

$$F = P(1+i)^N$$

$$₱500,000 = ₱290,000(1+i)^5$$

$$i = 11.51\%$$

34



Feb 12, 1998

35



Schedule expenses

36

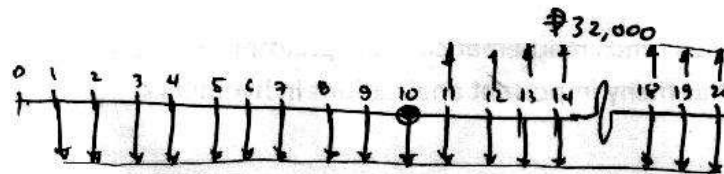


4

37

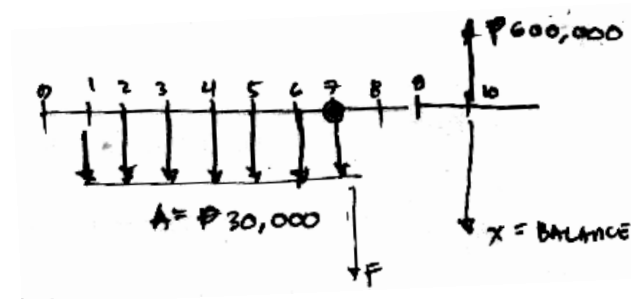


Examination, registration and license



$$A \left[\frac{(1.045)^{10} - 1}{0.045} \right] + A \left[\frac{1 - (1.045)^{-10}}{0.045} \right] = P32000 \left[\frac{1 - (1.045)^{-10}}{0.045} \right]$$

$$A = P12,534.4235$$



$$P600,000 = F(1.07)^4 + X$$

$$P600,000 = P30,000 \left[\frac{(1.07)^7 - 1}{0.07} \right] (1.07)^4 + X$$

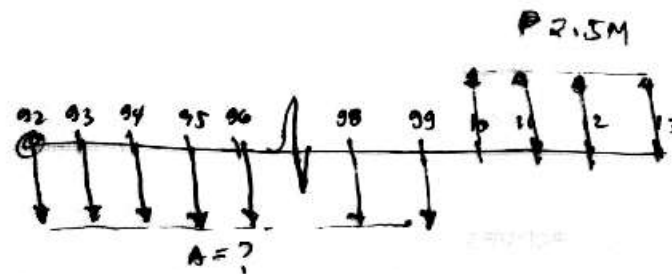
$$X = P259,690.3104$$

40



1793 and 9806

The total resistance is the sum of the individual resistors



$$A \left[\frac{(1.04)^{17} - 1}{0.04} \right] = 2.5M \left[\frac{1 - (1.04)^{-4}}{0.04} \right]$$

$$A = \text{P}382,940.5345$$

$i_n = 9\%$ compounded semi-annually

$$i = \frac{i_n}{m} = \frac{9\%}{2} = 4.5\% \text{ per 6 months}$$

$$P = A \left[\frac{1 - (1+i)^{-N}}{i} \right]$$

$$P180,000 = A \left[\frac{1 - (1.045)^{-10}}{0.045} \right]$$

$$A = P22,748.1879$$

hence;

$$P = P22,748.1879 \left[\frac{1 - (1.045)^{-6}}{0.045} \right]$$

$$P = P117,332.2525$$

$$F = A \left[\frac{(1+i)^N - 1}{i} \right]$$

$$₱6,432,000 = A \left[\frac{(1+0.09)^5 - 1}{0.09} \right]$$

$$A = ₱1,074,738.683$$

45



Portable boiler

46



Asbestos

47



Splash system

48



300 kW

49



35

50



Balance sheet and the profit and loss statements

$$C_0 = ₱22,000$$

$$C_L = ₱1,800$$

$$L = 8 \text{ years}$$

$$i_f = 7\%$$

$$i_i = 8\%$$

$$N = 8$$

Due to inflation

$$C_0' = C_0 (1 + i_f)^N = ₱22,000 (1 + 0.07)^8$$

$$C_0' = ₱37,800.0959$$

$$\text{Replacement Cost} = C_0' - C_L = ₱37,800.0959 - ₱1,800$$

$$\text{Replacement Cost} = ₱36,000.0959$$

Hence;

$$F = A \left[\frac{(1+i)^N - 1}{i} \right]$$

$$₱36,000.0959 = A \left[\frac{(1+0.08)^8 - 1}{0.08} \right]$$

$$A = ₱3,384.5404$$

$$F = A \left[\frac{(1+i)^N - 1}{i} \right]$$

$$₱130,000 = A \left[\frac{(1.025)^{10} - 1}{0.025} \right]$$

$$A = ₱11,603.6392$$

53

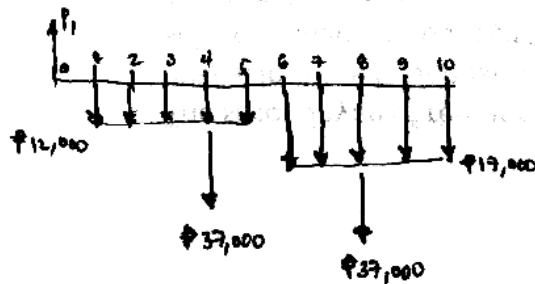


Labor cost

The acceleration of the gases through the engine

55

pH 10 to pH 11



$$P_1 = ₱12,000 \left[\frac{1 - (1.07)^{-5}}{0.07} \right] + ₱37,000 (1.07)^{-4} + ₱17,000 \left[\frac{1 - (1.07)^{-5}}{0.07} \right] (1.07)^{-5} + ₱37,000 (1.07)^{-8}$$

$$P_1 = ₱148,661.3587$$

then;

$$₱148,661.3587 = P_2 = A \left[\frac{1 - (1.07)^{-10}}{0.07} \right]$$

$$A = ₱21,166.0330$$

$$P = \text{P } 65,000 \left[\frac{1 - (1.10)^{-10}}{0.10} \right] + \text{P } 40,000 \left[\frac{1 - (1.10)^{-90}}{0.10} \right] (1.10)^{-10}$$

$$P = \text{P } 550,206.7571$$

$$d = \frac{C_0 - C_L}{L} = \frac{\text{P}520,300 - \text{P}60,000}{20}$$

$$d = \text{P}23,015$$

59

Lowest water space

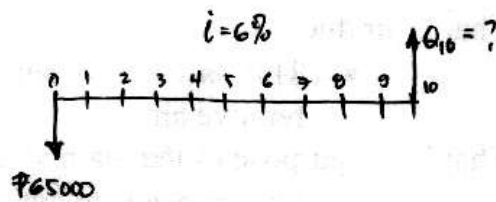
60

Is below the container surface due to cohesion

61

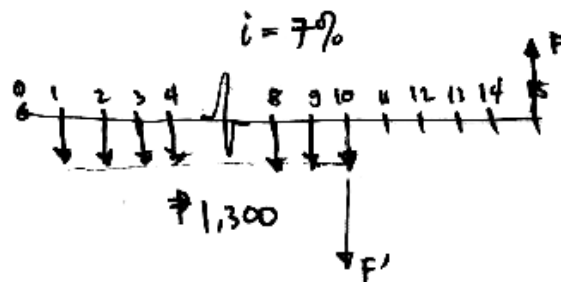


Surface tension



$$Q_{10} = ₱65,000 (1.06)^{10}$$

$$Q_{10} = ₱116,405.1003$$



$$F = F' (1.07)^5$$

$$F = \text{P}1,300 \left[\frac{(1.07)^{10} - 1}{0.07} \right] (1.07)^5$$

$$F = \text{P}25,191.7679$$

$$F = 2P$$

$$F = P(1+i)^N ; i = \frac{i_n}{m} = 1\% \text{ per month}$$

$$2P = P(1.01)^N$$

$$2 = 1.01^N$$

$$\ln 2 = N \ln 1.01$$

$$N = \frac{\ln 2}{\ln 1.01}$$

$$N = 69.6607$$

$$N = 70 \text{ months}$$

$$C_0 = \text{P}12,000$$

$$L = 20 \text{ years}$$

$$C_L = 0$$

Required: d_{11} using SYDM

$$d_n = \frac{\text{Reversed Digit}}{\text{Sum of Digit}} (C_0 - C_L)$$

$$\text{Reversed Digit} = L - n + 1 = 20 - 11 + 1 = 10$$

$$\text{Sum of Digit} = \frac{L}{2} (L + 1) = \frac{20}{2} (20 + 1) = 210$$

$$d_{11} = \frac{10}{210} (\text{P}12,000 - 0)$$

$$d_{11} = \text{P}571.4286$$

$$(\text{₱}1,200)(60) = A \left[\frac{(1.05)^{15} - 1}{0.05} \right]$$

$$A = \text{₱}3,336.6447$$

hence;

$$F_{12} = \text{₱}3,336.6447 \left[\frac{(1.05)^{12} - 1}{0.05} \right]$$

$$F_{12} = \text{₱}53,109.7959$$

$$d_n = \frac{\text{Reversed Depn}}{\text{Sum of Depn}} (C_0 - C_L)$$

$$\text{Reversed Depn} = L - n + 1 = 10 - 7 + 1 = 4$$

$$\text{Sum of Depn} = \frac{L}{2} (L + 1) = \frac{10}{2} (10 + 1) = 55$$

Book Value During 7th year

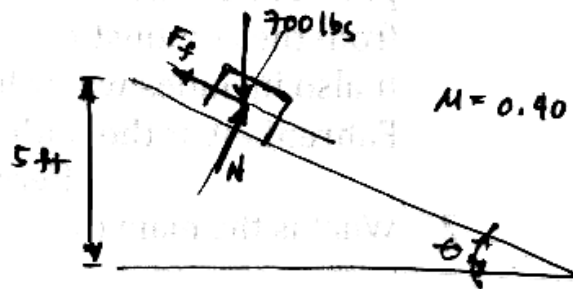
$$C_n = C_0 - D_n$$

$$C_n = C_0 - (d_1 + d_2 + d_3 + \dots + d_n)$$

$$C_7 = \text{P}8,000 - \frac{(\text{P}8,000 - \text{P}1,000)}{55} (4 + 5 + 6 + 7 + 8 + 9 + 10)$$

$$C_7 = \text{P}1,763.6364$$

68



Block will start to impede: $\theta = \phi$

$$\tan \theta = \frac{5 \text{ ft}}{\sqrt{9^2 + 5^2, \text{ ft}^2}}$$

$$\theta = 25.9032^\circ$$

and;

$$\tan \phi = \mu = 0.40$$

$$\phi = 21.8014^\circ$$

since: $\theta > \phi$; then the object will slide

therefore: Hold back

*n*th term:

$$a_n = a_1 + (n-1)d$$

Sum of Arithmetic Progression

$$S = \frac{n}{2} (a_1 + a_n)$$

$$a_{98} = 1.03 + (98-1)(-0.01)$$

$$a_{98} = 0.06$$

70

Characteristic length, mass flow rate per unit area, and absolute viscosity

71



Physical inventory

72



Prime cost

73



$\frac{2}{3} H$ from the vertex

$$PV = nRT$$

$$\rho = \frac{m}{V} = \frac{P}{RT}$$

$$\frac{P_1}{T_1 \rho_1} = \frac{P_2}{T_2 \rho_2}$$

$$\frac{2 \text{ atm}}{(105 + 460)^\circ \text{R} \left(0.095 \frac{\text{lb}}{\text{ft}^3} \right)} = \frac{P_2}{(260 + 460)^\circ \text{R} \left(0.281 \frac{\text{lb}}{\text{ft}^3} \right)}$$

$$P_2 = 7.5387 \text{ atm}$$

$$x^2 + y^2 + z^2 - 3x + 9y + 18z + 68 = 0$$

$$(x^2 - 3x + \frac{9}{4}) + (y^2 + 9y + \frac{81}{4}) + (z^2 + 18z + 81) = -68 + \frac{9}{4} + \frac{81}{4} + 81$$

$$\text{Thus; } r^2 = \frac{71}{2}$$

$$r = 5.9582$$

$$N_1 = 1800 \text{ rpm}$$

$$Q_1 = 600 \text{ gpm}$$

$$H_1 = 250 \text{ ft}$$

Due to change in piping system

$$H_2 = 380 \text{ ft}$$

from pump Affinity laws

$$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1} \right)^2$$

$$\frac{380 \text{ ft}}{250 \text{ ft}} = \left(\frac{N_2}{1800 \text{ rpm}} \right)^2$$

$$N_2 = 2219.1890 \text{ rpm}$$

$$\tan(20-3) = \frac{1}{\tan(50-9)}$$

Find D in degrees

using calculator or trial and error:

$$D = 14.57^\circ$$

78



0.1

79



Fusion welding

80

Lime soda treatment

81



It is valid only for potential flow

$\text{Assets} = \text{liabilities} + \text{owner's equity}$

83

$$J = \frac{\pi D^4}{32} = \frac{\pi (0.003 \text{ m})^4}{32} = 7.9522 \times 10^{-12} \text{ m}^4$$

$$\text{from; } S_s = \frac{T_c}{J}$$

$$80000 \text{ kPa} = \frac{T (1.5 \times 10^{-3} \text{ m})}{(7.9522 \times 10^{-12} \text{ m}^4)}$$

$$T = 4.2412 \times 10^{-4} \text{ kN-m}$$

2 complete revolutions:

$$(2)(360^\circ) \left(\frac{\pi}{180^\circ} \right) = 4\pi \text{ rad}$$

Hence;

$$\theta = \frac{TL}{JG}$$

$$4\pi = \frac{(4.2412 \times 10^{-4} \text{ kN-m}) L}{(7.9522 \times 10^{-12} \text{ m}^4) (38 \times 10^6 \text{ kPa})}$$

$$L = 8.9535 \text{ m}$$

*n*th term

$$a_n = a_1 + (n-1)d$$

$$a_{10} = 18.1 \text{ ft} + (10-1)(32.2)$$

$$a_{10} = 307.9 \text{ ft}$$

$$P_{\text{at least 2 heads in 4 toss}} = P_{2\text{head}} + P_{3\text{head}} + P_{4\text{head}}$$

$$\text{Use Probability of Repeated trials} = 4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 + 4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 + 4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0$$

$$P_{\text{at least 2 heads in 4 toss}} = \frac{11}{16}$$

$$F = \frac{A}{i} = \frac{\text{P}11,000}{0.12} = \text{P}91666.6667$$

then;

$$F = P(1+i)^N$$

$$\text{P}91666.6667 = \text{P}63,052 (1.12)^N$$

$$N = 3.3019 \text{ years}$$

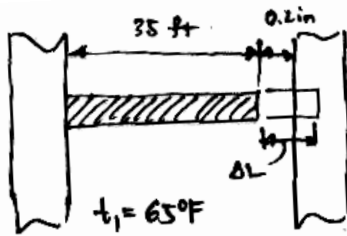
$$N = 3.5 \text{ years}$$

87



There must be an acceptance of the offer

88



$$t_1 = 65^\circ\text{F}$$

$$t_2 = 145^\circ\text{F}$$

$$\alpha = 6.8 \times 10^{-6} / ^\circ\text{F}$$

$$E = 32 \times 10^6 \text{ psi}$$

From Hooke's Law:

$$S_T = E \epsilon$$

$$S_T = \frac{E \Delta L}{L} ; \Delta L = \alpha L \Delta T$$

$$= (6.8 \times 10^{-6} / ^\circ\text{F})(35 \text{ ft})(145 - 65)^\circ\text{F}$$

$$\Delta L = (0.01904 \text{ ft}) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right)$$

$$\Delta L = 0.22848 \text{ in}$$

Hence;

$$\Delta L' = \Delta L - 0.2 \text{ in}$$

$$= 0.22848 \text{ in} - 0.2 \text{ in}$$

$$\Delta L' = 0.02848 \text{ in}$$

$$S_T = (32 \times 10^6 \text{ psi}) \left[\frac{0.02848 \text{ in}}{(35 \text{ ft}) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right)} \right]$$

$$S_T = 2,169.9048 \text{ psi compressive}$$

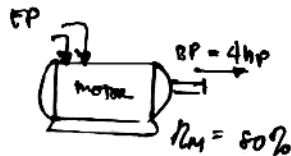
$$P = ₱7,900 + ₱1,200 \left[\frac{1 - \left(1 + \frac{0.07}{12}\right)^{-12}}{\frac{0.07}{12}} \right]$$

$$P = ₱21,768.5442$$

90



Professional Mechanical Engineer (PME)



Power cost:

1st 100 kw-hr ₱6.0/kw-hr

2nd 100 kw-hr ₱5.0/kw-hr

Remaining kw-hr ₱4.0/kw-hr

Running time: 10 days continuous operation

$$\eta_M = \frac{BP}{EP}$$

$$EP = \frac{BP}{\eta_M} = \left(\frac{4hp}{0.50} \right) \left(\frac{0.746 \text{ kw}}{1hp} \right) = 3.73 \text{ kw}$$

Energy Consumption

$$EP = (3.73 \text{ kw}) \left(10 \text{ days} \right) \left(\frac{24 \text{ hrs}}{1 \text{ day}} \right)$$

$$EP = 895.2 \text{ kw-hr}$$

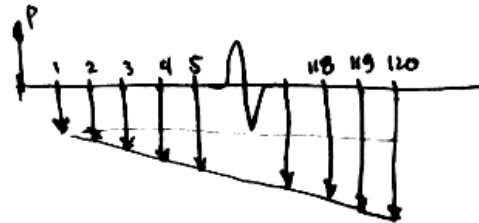
Thus;

$$1st \text{ 100 kw-hr} \rightarrow \frac{\text{₱6.0}}{\text{kw-hr}} (100 \text{ kw-hr}) = \text{₱600}$$

$$2nd \text{ 100 kw-hr} \rightarrow \frac{\text{₱5.0}}{\text{kw-hr}} (100 \text{ kw-hr}) = \text{₱500}$$

$$\text{Remaining kw-hr} \rightarrow \frac{\text{₱4.0}}{\text{kw-hr}} (895.2 \text{ kw-hr}) = \text{₱3,580.8}$$

$$\text{₱4,680.8}$$



$$i = \frac{10\%}{12} = 0.8333\% \text{ monthly}$$

Amount of debt;

$$P = P_{\text{annuity}} + P_{\text{gradient}}$$

$$= \text{₹}6,000 \left[\frac{1 - (1.008333)^{-120}}{0.008333} \right] + \frac{\text{₹}1,000}{0.008333} \left[\frac{(1.008333)^{120} - 1}{0.008333} - 120 \right] (1.008333)^{-120}$$

$$P = \text{₹}4,219,995.172$$

93



He must be Certified Plant Mechanic

94

Escalatory clause

95 to 97

$$\text{Total current Asset} = \text{Cash} + \text{A/R} + \text{MI} + \text{N/R}$$
$$\text{TCA} = \text{₱}60,000 + \text{₱}120,000 + \text{₱}210,000 + \text{₱}150,000$$

$$\text{TCA} = \text{₱}540,000$$

$$\text{Total current liabilities} = \text{N/P} + \text{A/P} + \text{S/P}$$

$$\text{TCL} = \text{₱}70,000 + \text{₱}140,000 + \text{₱}30,000$$

$$\text{TCL} = \text{₱}240,000$$

CURRENT RATIO

$$\text{CR} = \frac{\text{TCA}}{\text{TCL}} = \frac{\text{₱}540,000}{\text{₱}240,000}$$

$$\text{CR} = 2.25$$

98



Two speed drive motor

99



Net income before taxes/net sales

100



Gamma radiation